MAT2440 Practice Final Exam Halleck Spring 2019 Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* Book and notes are prohibited except for a single sheet (back and front) with hand-written formulae/notes. Submit formula sheet with your exam for up to 5 extra pts.
* You may write on test page. However, put all your work and answers into the blue book.
* No credit will be given for any answer that is not backed up with work.
* The use of any electronic devices except a graphing calculator is strictly prohibited.
1. Let C(x, y) be the statement “x finds y charming," where the domain for x and y consists of all people in the world. Use quantifiers to express each of the following statements.
	1. Everyone finds themselves charming.
	2. Someone finds Jerry charming.
	3. There is a person that finds everyone charming.
	4. If you find Max charming, then you will also find Gina charming.



1. Answer each of the following questions.
	1. Show that p ® q is logically equivalent to Ø q ® Ø p using a truth table.



* 1. Show that Ø p ® (q ® r) º q ® (p Ú r) without using a truth table.

Ø p ® (q ® r) º p Ú(Ø q Ú r) and q ® (p Ú r) º Ø q Ú( p Ú r)

Using commutative and associative laws for Ú, these expressions are equivalent (remove parentheses and rearrange terms).

* 1. Show that Ø (p ® q) ® Ø q is a tautology without using a truth table.

 Ø (p ® q) ® Ø q º (p ® q) Ú Ø q º (Ø p Ú q) Ú Ø q º Ø p Ú (q Ú Ø q) º Ø p Ú T º T

1. Prove that for all integers n, n is even if and only if 5n + 3 is odd.

(⇒) n even means n=2k, k ∈Z; 5n + 3 = 5(2k) + 3 = 10k + 2 + 1 = 2(5k + 1) + 1 so 5n + 3 is odd

(Ü) By contradiction: we assume that 5n +3 is odd but then we assume the opposite of the conclusion, namely that n is odd, i.e., n =2k+1. But then 5n + 3 = 5(2k + 1) + 3 = 10k + 8 = 2 (5k + 4) so 5n + 3 is even, which contradicts the assumption so in fact the conclusion must be true, namely n is even.

1. Translate the following specifications into English where:

F(p): “printer p is out of service"

B(p): “printer p is busy"

Q(j): “print job j is queued”

(a) ∃j Q(j) ® ∃p (F(p) Ú B(p)) If there is job queued, then some printer must be busy or out of service.

(b) ∀j Q(j) ® ∀p F(p) If all jobs are queued then all printers are out of service.

NOTE: assume that a print job that is being processed is no longer queued.

1. (a) Determine cardinality of the set A= $\{∅,\left\{a\right\}, \left\{∅,a\right\}\}$ as well as each member of set A.

|A|=3, |$∅$|=0, |$\left\{a\right\}$|=1, |$\left\{∅,a\right\}$|=2

(b) Draw the Venn diagram for the following combination of the sets A, B, and C.

$A∩(B-C)$ (shade *A* and $B-C$ as intermediate steps and use a legend)



1. Is it true that $A∩\left(B-C\right)=(A∩B)-\left(A∩C\right)$? Use a ~~truth~~ membership table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C | B-C | \* $A∩\left(B-C\right)$ | A$∩$B | A$∩$C | \*$(A∩B)-\left(A∩C\right)$ |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The two columns marked \* are identifical. ∴the identity being investigated is true.

1. Determine if the following functions are 1-1 and/or onto:
	1. $f:Z\rightarrow Z, f\left(x\right)=3x^{3}-2$

f is 1-1: Suppose f(a) = f(b), then $3a^{3}-2=3b^{3}-2$ and then $a^{3}=b^{3}$. We now apply the cube root to both sides. Cube root is a 1-1 function (its graph has a horizontal line only pass through 1 point), so this is legitimate. Hence we get a=b.

f is NOT onto: If we select almost any integer as an image, the preimage is not an integer. For instance, $3x^{3}-2=0$ means $x=\sqrt[3]{\frac{2}{3}}$ which is not an integer. The range is just a very limited set of integers:

{…., f(-1)=-5, f(0)=-2,f(1)= 1, f(2)=22,…} [For a function to be onto, the range must coincide with the codomain.]

* 1. $g:R\rightarrow Z, g\left(x\right)=\left⌊x/2\right⌋+6$

g is NOT 1-1: A horizontal line at any integer passes through more than one point. g(0)=g(.1)= 6.

g is onto: The set of even integers maps to all the integers: Given n∈Z, let m=2(n-6), then g(m)=n.

1. Use insertion sort with input 2, 1, 4, 3, 5, showing as separate steps the comparisons, rotations and insertions. You should have approximately 5+4+3+2+1 steps, each showing all or a portion of the list.

|  |  |  |  |
| --- | --- | --- | --- |
| working | Insert # | reserve | comments |
| 2\_ | 1 | 435 | 1<2 T |
| \_2 | 1 | 435 | rotate |
| 12 |  | 435 | insert |
| 12\_ | 4 | 35 | 4<1 F |
| 12\_ | 4 |  | 4<2 F |
| 124 |  | 35 | 1. insert
 |
| 124\_ | 3 | 5 | 1. 3<1 F
 |
| 124\_ | 3 | 5 | 1. 3<2 F
 |
| 124\_ | 3 | 5 | 1. 3<4 T
 |
| 12\_4 | 3 | 5 | 1. rotate
 |
| 1234 |  | 5 | 1. insert
 |
| 1234\_ | 5 |  | 1. 5<1 F
 |
| 1234\_ | 5 |  | 1. 5<2 F
 |
| 1234\_ | 5 |  | 1. 5<3 F
 |
| 1234\_ | 5 |  | 1. 5<4 F
 |
| 12345 |  |  | 1. insert
 |

1. The ISBN-10 of *Mathematical Modeling and Computer Simulation* is 0-534-Q8478-1, where *Q* is a digit. Find the value of *Q*. Use the Euclidean Algorthm (EA) to find the appropriate inverse.

1\*0+2\*5+3\*3+ 4\*4+5\*Q+6\*8+7\*4+8\*7+9\*8+10\*1=249+5\*Q≡0 mod 11 so 5\*Q≡-249 mod 11

-249=-23\*11+4 so -249 ≡4 mod 11 and 11=2\*5+1 or 11-2\*5=1 so inv(5) ≡ -2 ≡ 9 mod 11. Hence, Q≡ 9\*4 ≡ 36 ≡ 3 mod 11. As a check, 249+ 5\*3= 264 which is indeed 0 mod 11 (2-6+4=0)

1. If encryption function is *f* (*p*) = (7*p* + 13)**mod** 26, decrypt TZURCQKINZ: translate letters into #s, apply appropriate decryption function (use EA to find inverse of 7) & then translate #s back into letters.

26=3\*7+5, 7=1\*5+2 and 5=2\*2+1.

 **26**-3\***7**=**5**, **7**-1\***5**=**2** and **5**-2\***2**=1.

**5**-2\*(**7**-1\***5**)=1 or **5**-2\***7**+2\***5**=1 or -2\***7**+3\***5**=1

-2\***7**+3(**26**-3\***7**)=1 or -2\***7**+3\***26**-9\***7**=1 or 3\***26**-11\***7**=1

Hence inv(7) ≡ -11≡ 15 mod 26 As a check 7\*15=105≡ 1 mod 26

The decryption function is p=15(c-13)mod26 where c is the the number associated with the CT.

|  |  |  |  |
| --- | --- | --- | --- |
| **CT** | **NUM** | **DEC** | **PT** |
| T | 19 | 12 | M |
| Z | 25 | 24 | Y |
| U | 20 | 1 | B |
| R | 17 | 8 | I |
| C | 2 | 17 | R |
| Q | 16 | 19 | T |
| K | 10 | 7 | H |
| I | 8 | 3 | D |
| N | 13 | 0 | A |
| Z | 25 | 24 | Y |

By the way, the cyphertext’s last 2 letters have been changed. As Daniel had demonstrated in class, they were in error on the practice exam sheet.

1. Choose ONE of the following 2 INDUCTION proof problems:

(i)(a) Find formula for $\frac{1}{1∙2}+\frac{1}{2∙3}+\cdots +\frac{1}{n(n+1)}$ by examining values of this expression for small values of n

 (b) Prove the formula you conjectured in part (a).

(a) The first 3 sums are ½ , 2/3, ¾ (when simplified) so we conjecture the formula s(n)= n/(n+1)

(b) Basis step:1/(1\*2)= ½ = 1/(1+1)

Induction step: assume true for n=k, show true for n=k+1

s(k+1)=s(k)+1/((k+1)(k+2))= k/(k+1)+ 1/((k+1)(k+2)) =(k(k+2)+1)/ ((k+1)(k+2)) = (k^2+2k+1)/ ((k+1)(k+2)) = (k+1)^2/((k+1)(k+2)) = (k+1)/ (k+2) which completes the induction step

We conclude that the conjectured formula s(n)= n/(n+1) holds for all n>0

(ii) Prove that $3^{n}<n!$ if *n* is greater than 6.

Basis step: 3^7=2187<5040=7!

Induction step: assume true for n=k, show true for n=k+1

3^(k+1)=3\*3^k<3(k!)<(k+1)k!=(k+1)! (6<k, therefore 3<6<k<k+1)

Which completes the induction step

We conclude that the inequality holds for all n>6