1. (a) $\forall x \; C(x,x)$

(b) $\exists x \; C(x, \text{Jerry})$

(c) $\exists x \forall y \; C(x,y)$

(Note: $\forall y \exists x \; C(x,y)$ is incorrect!!)

(d) $\forall x \; C(x, \text{Max}) \rightarrow C(x, \text{Gina})$

2.

p	q	$p \to q$ *	$\neg q$	$\neg p$	$\neg q \to \neg p$ *
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

Since the 2 cols are marked * are identical, the respective statements are $\cong$

3. ($\Rightarrow$) n even, $n = 2k \Rightarrow 5n+3 = 5(2k)+3$
$= 10k+3 = 2(5k+1)+1$
$\therefore$ $5n+3$ is odd.

($\Leftarrow$) $5n+3$ odd $5n+3 = 2k+1$
$\Rightarrow 5n = 2k-2 = 2(k-1)$, $\Rightarrow 5n$ even
suppose n odd, then $5n$ is odd, contradiction
$\therefore$ n is even.

4. (a) $\forall x \exists y \; (\neg P \rightarrow \neg Q)$

(b) $xy - y^2 = 0 \quad (x-y)y = 0 \quad x = y$ or $y = 0$

Since $y \in \mathbb{Z}^+ \Rightarrow y \neq 0$, hence $y = x$

Statement is true since $x, y \in \mathbb{Z}^+$

Note: if $x \in \mathbb{Z}$ and $y \in \mathbb{Z}^+$, then this statement would NOT be true.

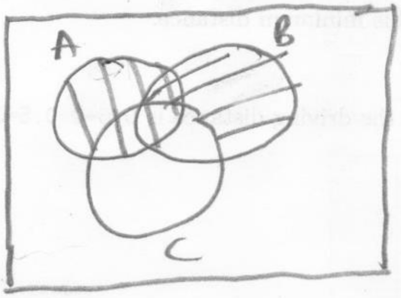
Let $x = -1$, then no appropriate y exists.

5. (a) If a job is queued, then some printer must be busy or out of servile

(b) If all jobs are queued then all printers are out of servile

6. (a) $|A| = 3$, $|\emptyset| = 0$, $|\{a\}| = 1$, $|\{\emptyset, a\}| = 2$.

(b)



A

$B - C$

$A \cap (B - C)$

(c)

A	B	B−A	A∪(B−A)	A∪B
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

Last 2 cols are identitical so the 2 sets are identical.

7. (a) 1-1 $3a^3 - 2 = 3b^3 - 2 \Rightarrow a^3 = b^3 \Rightarrow a = b$ so Y.

onto $3x^3 - 2 = y$ $x = \sqrt[3]{\frac{y+2}{3}}$ so Y

(b) 1-1 $\lfloor 1/2 \rfloor + 6 = \lfloor 0/2 \rfloor + 6 = 6$ so N

onto given $y \in \mathbb{Z}$ let $x = 2(y - 6)$ then $\lfloor x/2 \rfloor + 6 = y$ so Y

8.

261435
261435
216435
214635
214365
21435|6

214356
124356
124356
123456
1234|56

123456
123456
123456
123|456

123456
123456
12|3456
123456
|123456|

9. (a) count($x_1 \dots x_n$: integers)

num := 0
for i := 1 to n
  if $x_i < 3$ or $x_i > 8$
    num := num + 1
return(num) {num is # integers in list <3 or >8

(b) loc(x, $x_1 \dots x_n$: integers)
count := 0
while ($i \le n$ and count < 2)
  if $x_i > x$
    count := count + 1
  i := i + 1
If count = 2 return i − 1
else return 0

10. 00 01 02 …
10 11 12 …
20 21 22 …
30 31 32 …

formula: $(n \bmod 4, \lfloor n/4 \rfloor)$

11. $3\sum_{j=1}^{500} j - \sum_{j=1}^{500} 1 = 3\frac{500(501)}{2} - 500$

$$\boxed{\begin{aligned} &= 750(501) - 500 \\ &= 250(1503 - 2) \\ &= 250(1501) \end{aligned}}$$

12. (a) $\log n+1 - \log n$
$= \log \frac{n+1}{n} = \log 1 + \frac{1}{n}$
≈ 0 for large n

(b) $2(n+1) - 2n = 2$

(c) $(n+1)^4 - n^4 \approx 4n^3$ for large n

13. (a) $3^{n+1}/3^n = 3$ (b) $[2(n+1)]!/(2n)! = (2n+2)(2n+1)$
$\approx 4n^2$ for large n

14. still open for credit

15. Using Excel TZURQKINZRJQHTHCCHI

16.

n	1	2	3	4
$f(n)$	$\frac{1}{2}$	$\frac{1}{2}+\frac{1}{6}=\frac{2}{3}$	$\frac{2}{3}+\frac{1}{12}=\frac{3}{4}$	$\frac{3}{4}+\frac{1}{20}=\frac{4}{5}$

so conjecture $f(n)=\frac{n}{n+1}$

basis step $n=1$: $\frac{1}{1\cdot 2}=\frac{1}{2}$ ✓

induction step assume true for $n=k$ $f(k)=\frac{k}{k+1}$

show true for $k+1$

$$f(k)+\frac{1}{(k+1)(k+2)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)}$$

$=\frac{k+1}{k+2}$ which completes induction step.

$\therefore f(n)=\frac{n}{n+1}$

17. $3^n < n!$ $n \geq 7$ | basis step: $2187 = 3^7 < 7! = 5040$ ✓

inductive step assume $3^k < k!$

$$3^{k+1} = 3\cdot 3^k \underset{\text{ind. assum}}{<} 3\cdot k! \underset{3<7\leq k<k+1}{<} (k+1)\cdot k! = (k+1)!$$

$\therefore 3^n < n!$