Review for Exam 2 MAT 2440 Halleck Sp 18

1. Let $A = \{3, 4, 5\}$ and $B = \{0, 1, 2, 3, 6\}$. Assuming that the universal set $U = \{0, 1, 2, 3, 4, 5, 6\}$ list the elements of each of the following sets.

   (a) $\overline{A} \cap B$

   (b) $A \cup B$

   (c) $B - A$

   (d) $\overline{A \cup B}$

2. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

3. Draw the appropriate Venn diagram for each of these combinations of sets A, B, and C.

   (a) $(A - B) \cap C$

   (b) $A \cap B \cap C$

   (c) $A \cup \overline{C}$

   (d) $\overline{A \cup (B \cap C)}$

4. Determine if the following functions are one-to-one and/or onto.

   (a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = -2x + 7$

   (b) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m, n) = |m|$

   (c) $f : \mathbb{Z} \to \mathbb{Z}$ where $f(n) = n^2$

5. Suppose that $g : A \to B$ and $f : B \to C$ are functions.

   (a) If g and f are onto, then $f \circ g$ is onto.

   (b) If g and f are one-to-one, then $f \circ g$ is one-to-one.

6. How many bytes of data are required to encode n bits of data where n equals 7? 17? 1001? (1 byte = 8 bits)

7. Find the first five terms a_0, a_1, a_2, a_3 of each sequence:

   (a) $a_n = n^3 + \frac{2}{n+1}$ for $n \geq 0$ and

   (b) $a_n = (-2)^n$ for $n \geq 0$

8. Find the terms a_1, a_2, a_3 for the sequence given by the following recurrence relation: $a_n = 2n + a_{n-1}$ for $n \geq 1$ and $a_0 = 2$.

   Using Microsoft Excel, find the 101st term (=a_{100})

9. Using Microsoft Excel, find the following sums.

(Hint: for the second one, create a 2-dimensional array).

(a) $\sum_{j=0}^{4}(1+(-2)^j)$

(b) $\sum_{i=1}^{3}\sum_{j=1}^{3}(i-j)$

10. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a bijection between the set of positive integers and that set.

    (a) Integers not divisible by 3

    (b) Real numbers with decimal representations containing only 0s and 1s.

    (c) $\{1,2\} \times \mathbb{N}$.

11. Write the pseudocode for an algorithm that takes a list of n integers and produces as output the sum of the numbers in the list.

12. Write the pseudocode for an algorithm that takes in a list of n integers and produces as output the smallest difference between consecutive numbers in the list.

13. Write the pseudocode for an algorithm that finds both the largest and smallest integers in a finite sequence of integers.

14. Use bubble sort to sort 3, 1, 5, 7 4, showing the lists obtained at each step.

15. Use insertion sort to sort 3, 1, 5, 7 4, showing the lists obtained at each step.

16. Use the greedy algorithm to make change using quarters, an 18 cent coin, dimes, nickels and pennies for 77, 43, and 24 cents.