

MAT 2440 Hallett Spg 2018
 Practice Exam 1 Solutions

1. $(\neg p \wedge (p \vee q)) \rightarrow q$

Approach 1, change $a \rightarrow b$ to $\neg a \vee b$

$$\neg(\neg p \wedge (p \vee q)) \vee q = p \vee \neg(p \vee q) \vee q$$

$$= (p \vee q) \vee \neg(p \vee q) \text{ which is clearly a tautology}$$

[let $r = p \vee q$ then $= r \vee \neg r$]

Approach 2, truth table

p, q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
11	0	1	0	1
10	0	1	0	1
01	1	1	1	1
00	1	0	0	1

2. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
 $(\neg p \vee q) \wedge (\neg p \vee r) \equiv \neg p \vee (q \wedge r)$

Think:
 try
 identity
 proof!

alternatively use truth table
 However, there are 8 inputs so much more work!

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3. Since only 4 inputs, lets use truth table.

pq	$p \leftrightarrow q$	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
11	1	1	0	0	0	1
10	0	0	0	1	0	0
01	0	0	1	0	0	0
00	1	0	1	1	1	1

4. a) $(p \rightarrow q) \rightarrow r \equiv (\neg p \vee q) \rightarrow r \equiv \neg(\neg p \vee q) \vee r \equiv (p \wedge \neg q) \vee r$

b) $p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r) \equiv \neg p \vee \neg q \vee r$

To show not equivalent, want same input, diff. output.
If $r = 1$, then clearly both are 1, so set $r = 0$

Both 000 and 010 make a) = 0 and b) = 1

5. a) There do not exist ignorant professors.

b) If there is a vain, knowledgeable person,
then there is a professor.

c) Either every person is vain or
if every person is ignorant,
then there are no professors.

6. a) $\forall x Q(x) \rightarrow R(x)$

b) $\neg \exists x P(x) \wedge Q(x) \equiv \forall x [\neg P(x) \vee \neg Q(x)]$

c) $\exists x [P(x) \wedge R(x)]$

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7. (a) $\neg(\forall x \exists y P(x,y)) \equiv \exists x \forall y \neg P(x,y)$

(b) $\neg[\exists y (Q(y) \wedge \forall x \neg R(x,y))]$

$$\equiv \forall y \neg [Q(y) \wedge \forall x \neg R(x,y)]$$

$$\equiv \forall y \{ \neg Q(y) \vee \neg [\forall x \neg R(x,y)] \}$$

$$\equiv \forall y [\neg Q(y) \vee \exists x R(x,y)]$$

8. (a) Every comedian is funny.

(b) Every person is a funny comedian.

(c) There is at least one comedian who is not funny.

9. (a) valid, modus tollens (contrapositive)

(b) fallacious, affirming conclusion (converse)

(c) valid, simplification gives p, q

modus ponens gives r, q

conjunction gives $r \wedge q$

10. (a) valid: Universal modus ponens

(b) fallacious: deny hypothesis (inverse)

11. (\Leftarrow) (direct) $n^{\text{even}} \Rightarrow 2k \Rightarrow n^2 = 4k^2 = 2(2k^2) \Rightarrow n^2 \text{ even}$

(\Rightarrow) (contradiction) $n^{\text{odd}} \Rightarrow n = 2k+1 \Rightarrow n^2 = (4k^2 + 4k) + 1 \Rightarrow n^2 \text{ odd} \Rightarrow \Leftarrow$

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12. $n \in \mathbb{Z} \wedge n^3 + 5 \text{ odd} \Rightarrow n \text{ even}$

(a) $n \text{ odd} \Rightarrow n = 2k+1 \Rightarrow n^3 + 5 = (2k+1)^3 + 5$

$$= 8k^3 + 12k^2 + 6k + 1 + 5$$

$$= 2(4k^3 + 6k^2 + 3k + 3) \text{ even} \Rightarrow \text{even}$$

$$\begin{array}{cccc} & & & 1 \\ & & & 1 \\ & & 1 & 1 \\ & 1 & 2 & 1 \\ 1 & 3 & 3 & 1 \end{array}$$

(b) same p except last line has \neq at end.