# MAT 2630 Halleck Fall 2015 Practice Exam 3

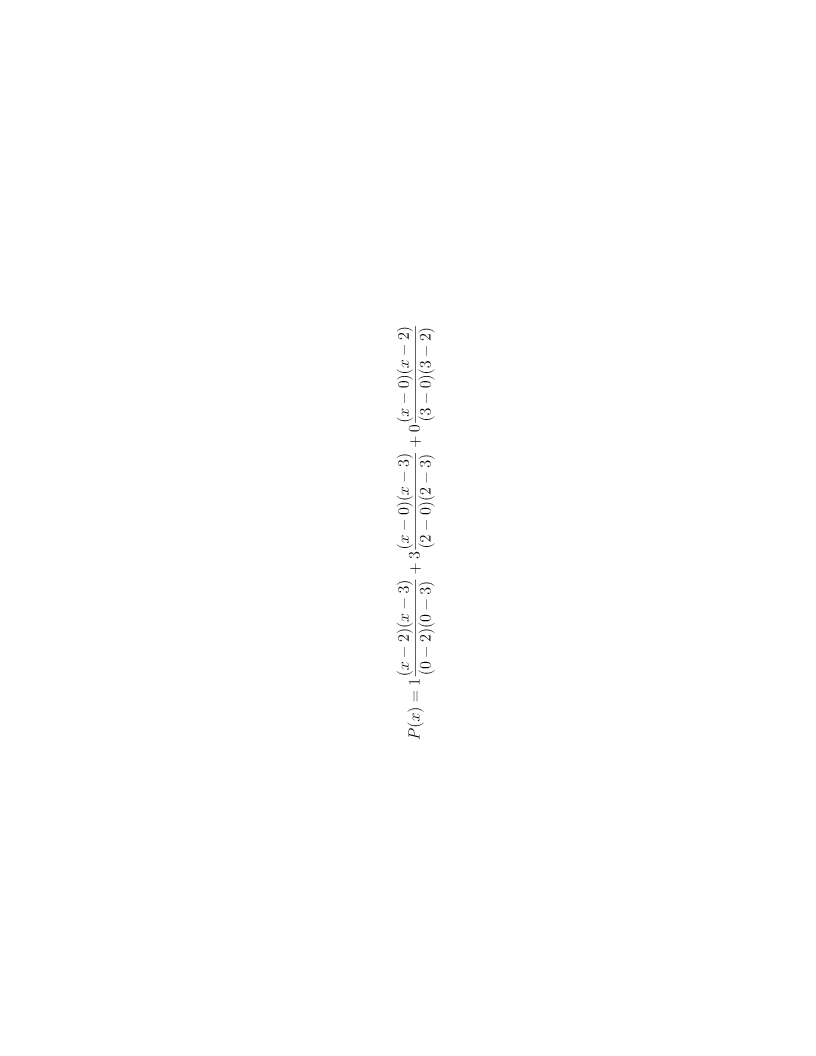
**REMINDER: your 2 page (front and back) 1 sheet hand-written set of formulas and notes will be 10% of your grade.**

**Please do as much of the exam as you can by hand. However, you may use a calculator if you need it. The actual exam will consist of questions similar to 5 of the ones that you see below. Each question will be worth 18%.**

1. A) Use Lagrange interpolation to find a polynomial that passes through the points (0,−2), (2,1), (4,4).

B) Use Newton’s divided differences to find the interpolating polynomial

(You can leave each in raw form. You do not need to check that they are equivalent.)



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | -2 |  |  |  | 0 | -2 |  |  |
|  |  | (1-(-2))/2=3/2 |  |  |  |  | 3/2 |  |
| 2 | 1 |  | (3/2-3/2)/4=0 |  | 2 | 1 |  | 0 |
|  |  | (4-1)/2=3/2 |  |  |  |  | 3/2 |  |
| 4 | 4 |  |  |  | 4 | 4 |  |  |

So P(x)=-2+3/2x

1. Find the one-piece Bézier curve (x(t ),y(t)) defined by the given four points (1,2), (1,3), (2,3), (2,2). Determine the points corresponding to t=.25, t=.5 and t=.75. Use them to sketch the curve on graph paper. Use 5 boxes is one unit.

 so 



1. Use the three-point centered-difference formula for the second derivative to approximate f ″(0), where f (x) = cos x, for (a) h = 0.1 (b) h = 0.01 (c) h = 0.001. Find a bound on the approximation error. Compare with the actual error.





1. Apply the composite Simpson’s Rule with m = 1 and 2 panels to the integrals, and report the errors:

1 panel:



2 panels:



These are live tables from Excel and you can open them up (right click) and play with them in Excel. They can easily be manipulated for any number of panels.

1. Apply Euler’s Method with step size h = 1/4 to the IVP y ¢= 2(t + 1)2y; y(0)=1 on the interval [0,1]. List the wi, i = 0, . . . , 4, and find the error at t = 1 by comparing with the correct solution. If the step is halved, by how about much will the error decrease?

 Let then Substituting in the initial value, we have 



The above table again is from a live Excel file that can be opened using a right click and then played with.

The answer is that we should expect ½. Surprisingly the error is not halved when going from 4 to 8.

As seen from the chart below, the error goes from -92 (4 steps) to -77 (8 steps). However, later on, the errors get closer and closer to being halved. The moral of the story is that for small h, we get the expected behavior, but for large h, like ¼, 1/16, , even 1/32, you can’t expect the expected behavior with high growth functions (this function is **triply** exponential).

log2(steps) y approx y exact error

1.0000 6.5000 106.3427 -99.8427

2.0000 14.3718 106.3427 -91.9709

3.0000 28.9991 106.3427 -77.3436

4.0000 49.1772 106.3427 -57.1654

5.0000 69.3665 106.3427 -36.9761

6.0000 84.8163 106.3427 -21.5264

7.0000 94.6417 106.3427 -11.7009

8.0000 100.2297 106.3427 -6.1130

9.0000 103.2166 106.3427 -3.1261

10.0000 104.7617 106.3427 -1.5810

11.0000 105.5476 106.3427 -0.7950

12.0000 105.9440 106.3427 -0.3987

13.0000 106.1431 106.3427 -0.1996

14.0000 106.2428 106.3427 -0.0999

15.0000 106.2927 106.3427 -0.0500

16.0000 106.3177 106.3427 -0.0250

17.0000 106.3302 106.3427 -0.0125

18.0000 106.3364 106.3427 -0.0062

19.0000 106.3396 106.3427 -0.0031

20.0000 106.3411 106.3427 -0.0016

Matlab code:

f=@(t,y) 2\*(t + 1)^2\*y;

tab=[];

ye=exp(2/3\*((1+1)^3-1));

for i=1:20

ya=eul(f,0,1,1,2^i);

tab=[tab;[2^i,ya,ye,ya-ye]];

end

disp(' log2(steps) y approx y exact error')

disp(tab)

1. Apply Midpoint Method with step size h = 1/2 to the IVP y ¢= 2(t + 1)2y; y(0)=1 on the interval [0,1]. List the wi, i = 0, . . . , 2, and find the error at t = 1 by comparing with the correct solution. If the step is halved, by how about much will the error decrease?

There will be an extra credit diffeq problem with midpoint (like this)or trapezoid methods called for.