MAT 2630 Halleck Fall 2015 Practice Exam 2
REMINDER: your 2 page (front and back) 1 sheet hand-written set of formulas and notes will be $\mathbf{1 0 \%}$ of your grade.
Please do as much of the exam as you can by hand. However, you may use a calculator if you need it. The actual exam will consist of questions similar to 5 of the ones that you see below. Each question will be worth $18 \%$.

1. Solve the system $A x=[2 ; 4 ; 6]$ by finding the $L U$ factorization for the matrix $A$ below and using the two-step back substitution.
$\left[\begin{array}{lll}2 & 2 & 3 \\ 4 & 2 & 0 \\ 4 & 4 & 2\end{array}\right]$
2. For the system of equations: $x_{1}-2 x_{2}=3,3 x_{1}-4 x_{2}=7$
a. Find the condition number for the coefficient matrix.
b. Solve the system exactly.
c. Find the forward and backward errors and error magnification factor for the approximate solution $[-2,-3]$.
3. Find the $P A=L U$ factorization for the matrix $A$ below and check by matrix multiplication.
$\left[\begin{array}{ccc}1 & 2 & -3 \\ 2 & 4 & 2 \\ -1 & 0 & 3\end{array}\right]$
4. Rearrange the equations to form a strictly diagonally dominant system. Apply two steps of the Gauss-Seidel Method from starting vector [0;0;0].
$u-8 v-2 w=1$
$u+v+5 w=4$
$3 u-v+w=-2$
5. Verify that the symmetric matrix $A$ below is positive definite. Find the Cholesky factorization $A=R^{\top} R$ :
$\left[\begin{array}{ccc}1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2\end{array}\right]$
6. Find the best line through $(0,0),(1,3),(2,3),(5,6)$, and find the RMSE. Graph the points as well as the solution. Verify that $\sum_{i=1}^{4}\left(c_{1}+c_{2} t_{i}-y_{i}\right)^{2} / \sqrt{4}=R M S E$.

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7. Find the best parabola through $(0,0),(1,3),(2,3),(5,6)$, and find the RMSE. Graph the points as well as the solution. Find $\sum_{i=1}^{4}\left(c_{1}+c_{2} t_{i}+c_{3} t_{i}^{2}-y_{i}\right)^{2} / \sqrt{4}$. Is it the same as the RMSE?

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