MAT 2630 Halleck Fall 2015 Practice Exam 1 v3

You will have 1 hour to do part 1 and 40 minutes to do part 2.

REMINDER: your 2 page (front and back) 1 sheet hand-written set of formulas and notes will be 10% of your grade. Make sure that you turn it in as part of the 2nd part of your exam.

Part 1, no MATLAB or computer is allowed beyond a graphing calculator.

1. a) Rewrite the polynomial in nested form:
   $P(x) = -3x^4 - 6x^3 + 18x^2 + 12x$

   b) In nested form evaluate at $x = -\frac{1}{2}$. Show 4 intermediate calculations and answer as a fraction.

2. How many additions and multiplications are required to evaluate the polynomial
   $Q(x) = 3x^6 - 3x^4 + 9x^2 + 1$ in nested form by thinking of
   a) Q(x) as a polynomial in x

   b) Q(x) as a polynomial in x^2

3. Find binary representations of the base 10 numbers (you don't need to convert to floating point)
   a) 93

   b) 11.2

4. Convert the base 10 numbers to binary and express as a binary floating point number fl(x) by using Rounding to Nearest Rule (but not as the machine represented number):
   a) 9.5

   b) 9.6

5. Do the following sums using IEEE Rounding to Nearest Rule. Express as a binary floating point number fl(x)
   a) (1+(2^-51+2^-52+2^-54))-1

   b) (1+(2^-51+2^-52+2^-54+2^-60))-1

6. Use Gaussian elimination to solve the 3x3 system. Describe the elementary row operation (or operations) used at each step. After the system is in triangular form, keep track of the arithmetic operations performed and verify that they total 9.

$$
\begin{aligned}
x + 2y - z &= 2 \\
y + 5z &= 6 \\
3y + z &= 4
\end{aligned}
$$

7. Suppose that your computer takes 0.005 seconds to perform back substitution for a system whose matrix is 5000x5000.
   a) How long will it take approximately to do Gaussian elimination for this system?

   b) By doing the most minimum of calculations, determine approximately how long it will take perform back substitution on this same computer for a system that is represented by a matrix of size 10000x10000.

   c) By doing the most minimum of calculations, determine approximately how long it will take to perform Gaussian elimination for a system that is represented by a matrix of size 10000x10000.

Part 2 Using MATLAB (you may use templates that you create from a USB flash drive). Please turn in any code you use to Blackboard at the end of the period AS A SINGLE script m-FILE.

8. As a preliminary, bring all terms $x^4 = x^3 + 10$ to one side and graph the resulting function from x = −2 to x = 3. Provide a quick sketch on the axes below and label any zeros.

   a) State the IVT.

   b) Using the IVT as a tool, find an interval of length 1 that contains the negative root of $x^4 = x^3 + 10$.

   c) Using the IVT as a tool, find an interval of length 1 that contains the positive root of $x^4 = x^3 + 10$.

   d) How many iterations are approximately required using the bisection method to find a solution to within 10^{-10} of the actual root?

9. Take the equation $x^4 = x^3 + 10$ and solve for x by subtracting x^3 and factoring out x^3 from the left side, dividing both sides by the binomial factor and taking the cube root. Would Fixed-point iteration converge given that the root is approximately 2.1? If so, would its convergence be better or worse than bisection?

10. We will now discuss Newton's method to find a root for $x^4 = x^3 + 10$.
    a) Apply 2 steps of Newton's method with initial guess $x_0 = 2$. Don't use any of the m-functions that have been provided for this purpose. Instead use MATLAB essentially as a calculator. Show all the calculations.

    b) How fast will Newton's method converge? Justify your answer.