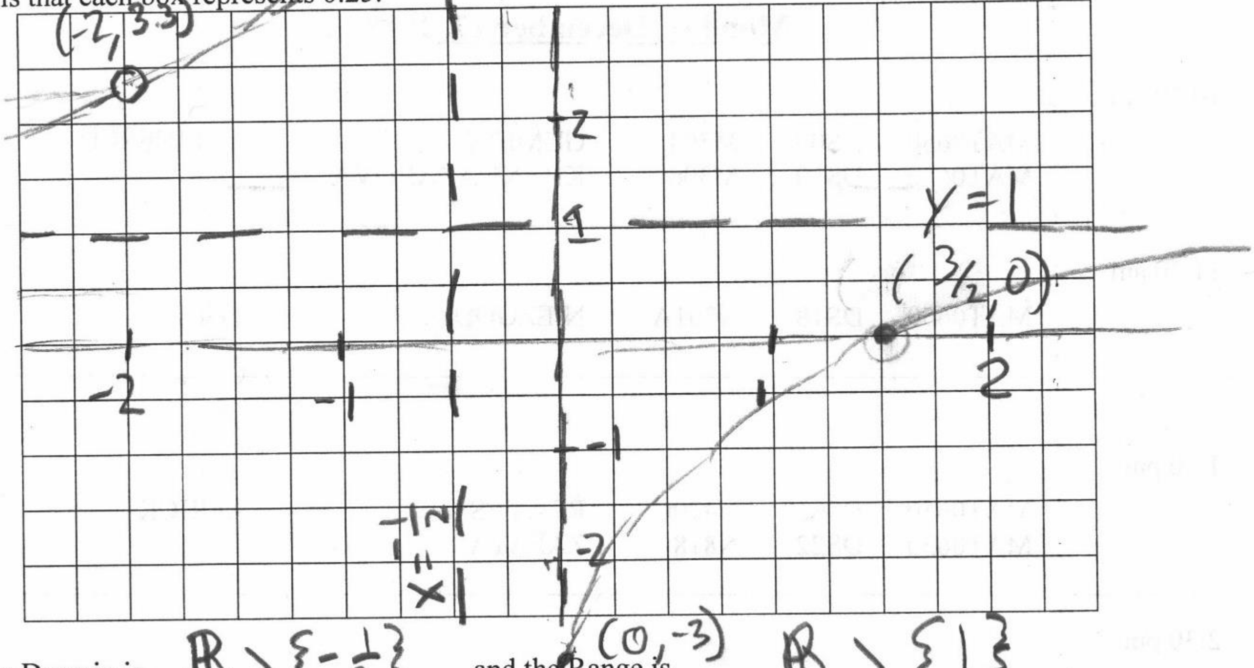
Practice Exam 2 Halleck **MAT 1375** **Spring 2019**

Actual exam will have 6 questions similar to what follows (6, 7 & 8 may also use Tan). Each problem will be worth 15 points. 10 points will be based on solutions that you provide to practice exam. Use complete sentences to answer word problems.

For graphs, find & label with coordinates all x/y-intercepts, holes & relative extrema. Draw asymptotes as dashed lines and label with their equations.

$f(-2) = \frac{-7}{-3} = \frac{7}{3}$

(1) a) On grid below, sketch complete graph of $f(x) = \frac{(2x-3)(x+2)}{(2x+1)(x+2)}$. Use for scale on the x-axis that each box represents 0.25.



b) The Domain is $\mathbb{R} \setminus \{-\frac{1}{2}\}$ and the Range is $\mathbb{R} \setminus \{1\}$.

c) Translate the inequality $\frac{(2x-3)(x+2)}{(2x+1)(x+2)} > 0$ into a graph statement & use the graph to solve. Write the answer in interval notation.

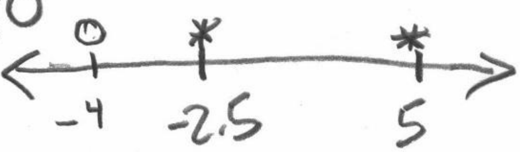
Find all x-values of where the graph for f(x) is above x-axis.

$(\frac{3}{2}, \infty)$

(2) Use any method to solve $\frac{1}{2x+5} \leq \frac{3}{x-5}$.

$$\frac{1}{2x+5} - \frac{3}{x-5} = \frac{(x-5)-3(2x+5)}{(2x+5)(x-5)} = \frac{x-5-6x-15}{(2x+5)(x-5)}$$

$$= \frac{-5x-20}{(2x+5)(x-5)} = \frac{-5(x+4)}{(2x+5)(x-5)} \leq 0$$



Test points x	-5	-3	0	6
f(x)	.1	-.625	.8	-2.941 <Y
verdict	N	Y	N	[-4, -2.5) ∪ (5, ∞)

(3) a) The amount of butonium in the US arsenal is 1,000 kg and is decreasing exponentially at 3.5% per year. What is the half life of this butonium? In how many years will there only be 1 kg left?

$y = 1000(.965)^t = 500$

$(.965)^t = \frac{1}{1000}$

$t = \frac{-3}{\log(.965)} = 193.89$ yr

In ~194 yrs, only 1 kg will be left

$(.965)^t = \frac{1}{2}$

$t \ln .965 = \ln \frac{1}{2}$

Half life is ↓

$t = \frac{\ln\frac{1}{2}}{\ln .965} = 19.46$ yr

b) In 2017, the population of a Austin, TX was 950,000 people, and is growing at a rate of 1.5% per year. What will the population be in 2020? In what year will the population surpass 1 million? To be able to answer this question, what assumption are you making?

$y = 950k(1.015)^t$

Population in 2020 is ↓

$y(3) = 950k(1.015)^3 = 964,535$

$950k(1.015)^t = 1000k$

$(1.015)^t = \frac{1000}{950} = \frac{100}{95} = \frac{20}{19}$

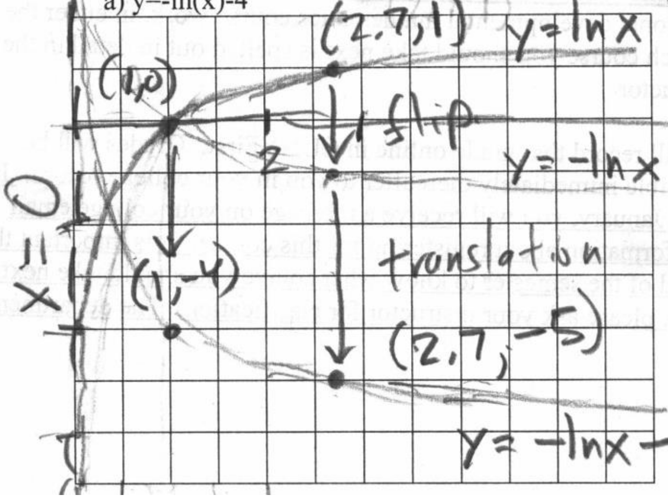
$t = \ln(20/19) / \ln(1.015) = 3.445$ yr.

In 2023, population will surpass 1 million.

We assume figures are beginning of each yr

(4) For BOTH parts a) and b), describe the transformation(s) from the basic shape & then sketch graphs on the same axes of the function and its basic shape. Use at least 2 guide points and show using the arrows the effect of the transformations(s) on these guide points. Find the domain and range.

a) y=-ln(x)-4



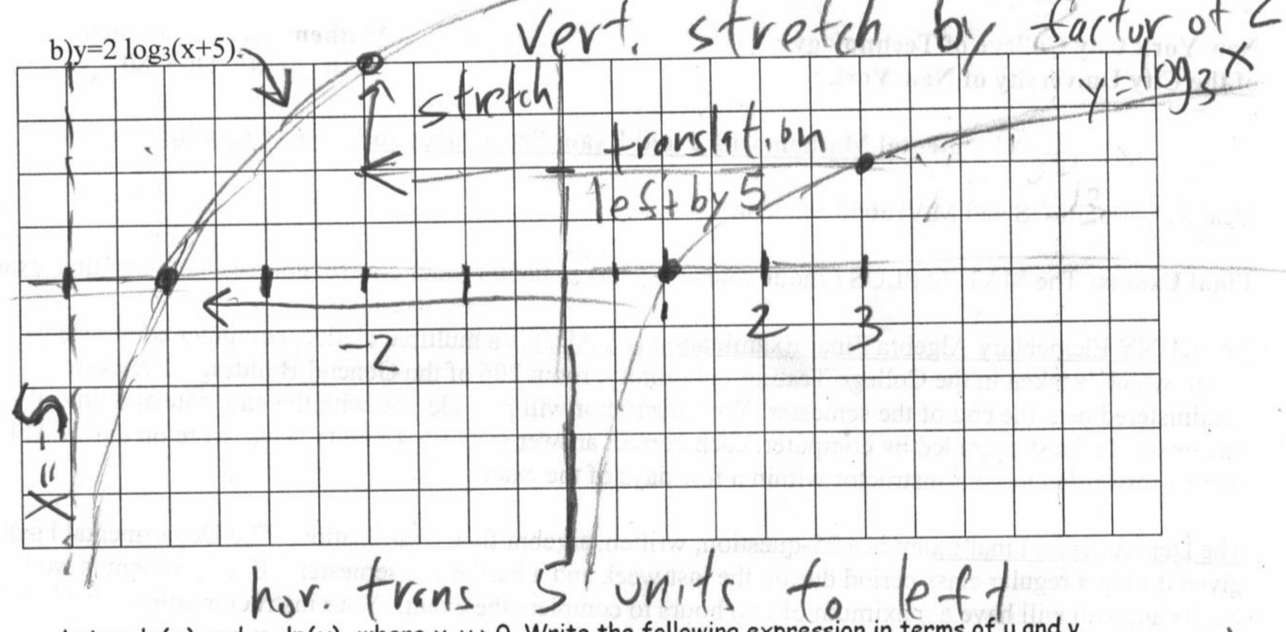
vert flip over x axis

translation down by 4

$D = (0, \infty)$

$R = (-\infty, \infty)$.

b) $y=2\log_3(x+5)$.



(5) Let $u=\ln(x)$ and $v=\ln(y)$, where $x, y > 0$. Write the following expression in terms of u and v.

b) $\ln\left(\frac{\sqrt{x^5}}{y^3}\right)$

$\ln x^{\frac{5}{2}} - \ln y^3$

$= \frac{5}{2}\ln x - 3\ln y$

$= \frac{5}{2}u - 3v$

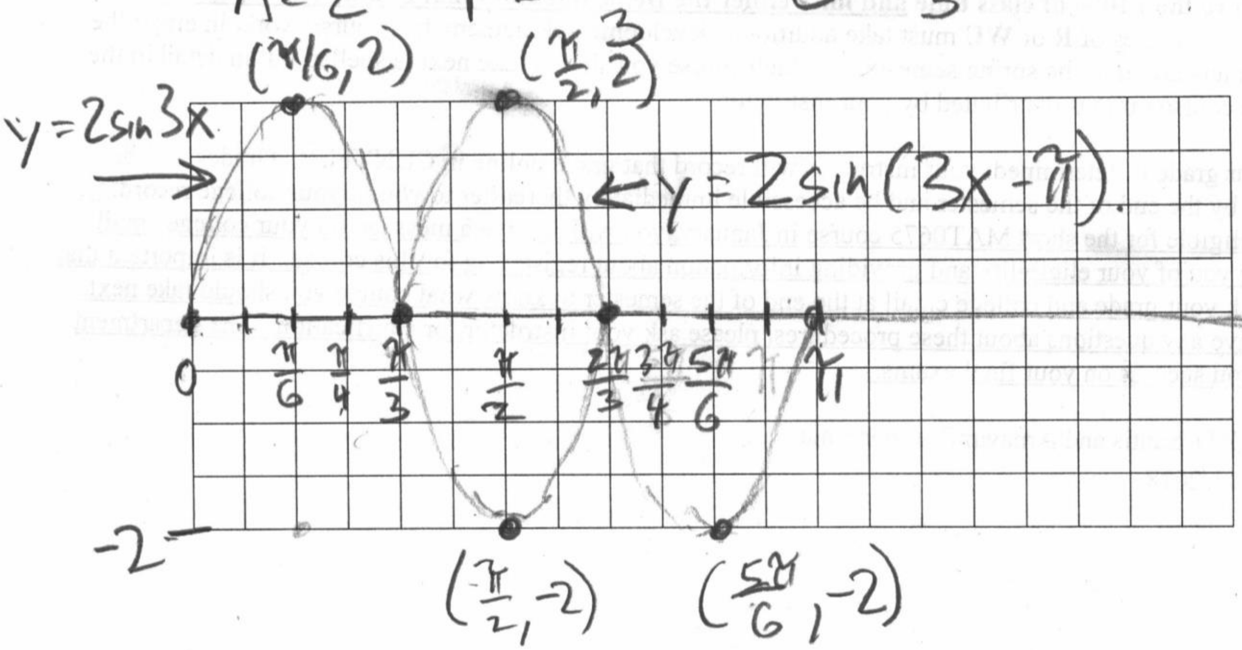
c) $\ln\left(\sqrt{x^3\cdot\sqrt[4]{y}}\right)$ $= \frac{1}{2}\left(\ln x^3 + \ln y^{\frac{1}{4}}\right)$

$= \frac{1}{2}\left(3\ln x + \frac{1}{4}\ln y\right)$

$= \frac{1}{2}\left(3u + \frac{1}{4}v\right)$

(6) On grid below, sketch complete graphs of both $y=2\sin(3x-\pi)$ and $y=2\sin(3x)$ over 1 period (treat $y=2\sin(3x)$ as the basic shape). Use for scale on the x-axis that each box is $\pi/12$ and on y-axis that each box represents 1/2. Find amplitude A, period P & phase φ.

$A=2$ $P=\frac{2\pi}{3}$ $\varphi=\frac{\pi}{3}$



(7) On grid below, sketch complete graph of $y=-4\cos(\pi x/3)+1$ & $y=-4\cos(\pi x/3)$ over 1 period (treat $y=-4\cos(\pi x/3)$ as basic shape). Use for x-axis scale: each box = 1/2. Find the exact zeros with the aid of a unit circle (will be an expression in terms of an inverse trig function), but on graph, label with an approximation. Find amplitude A, period P & phase φ.

$2\pi-\cos^{-1}(\frac{1}{4})$

$\cos\theta=\frac{1}{4}$

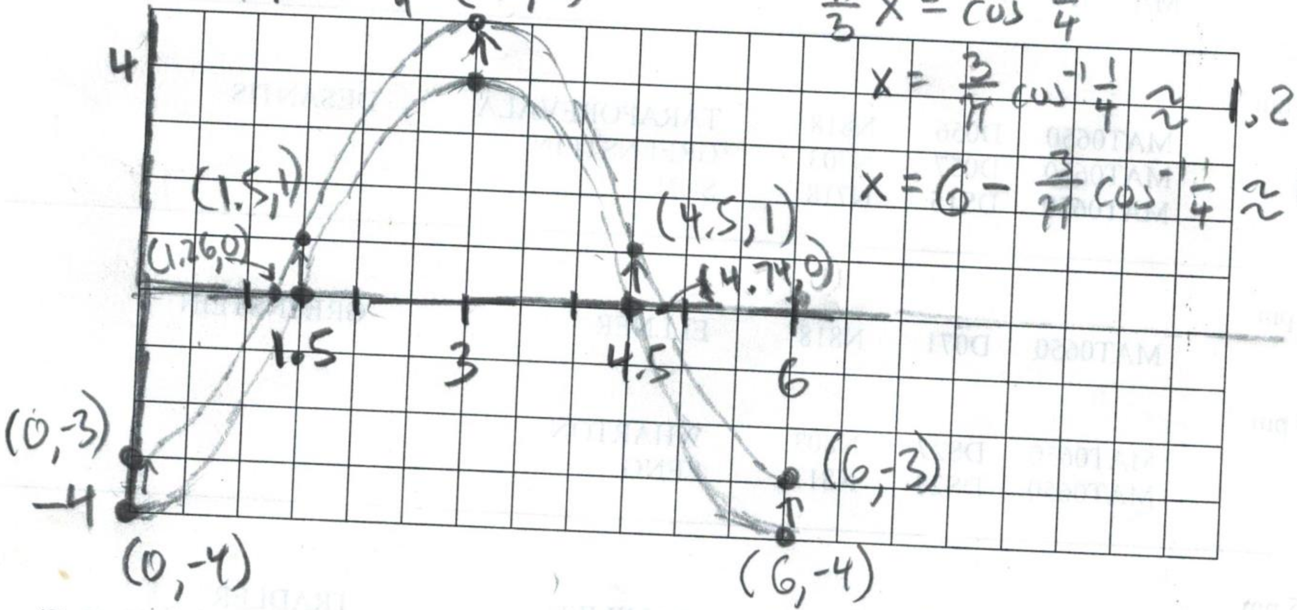
$\theta=\cos^{-1}\frac{1}{4}$

$A=4,\ P=\frac{2\pi}{\pi/3}=6,$ φ = 0

$\frac{\pi}{3}x=\cos^{-1}\frac{1}{4}$

$x=\frac{3}{\pi}\cos^{-1}\frac{1}{4}\approx 1.26$

$x=6-\frac{3}{\pi}\cos^{-1}\frac{1}{4}\approx 4.74$

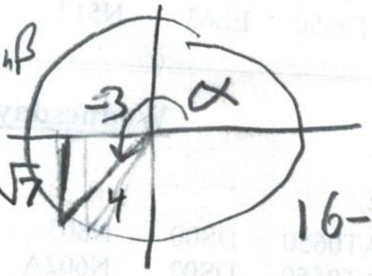


(8) Given that α is in quadrant 3 & cos(α)=−3/4, and that β is in quadrant 2 & sin(β)=5/8, find (on exam, may instead be double, half or sum):

a. sin(α−β) $=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$=\frac{-\sqrt{7}}{4}\cdot\frac{-\sqrt{39}}{8}-\frac{-3}{4}\cdot\frac{5}{8}$

$=\frac{\sqrt{273}+15}{32}$



$39\times7=273$

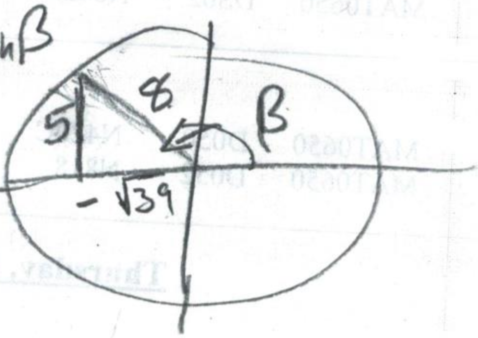
$16-9=7$

$64-25=3$

b. cos(α−β) $=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$=-\frac{3}{4}\cdot\frac{-\sqrt{39}}{8}+\frac{-\sqrt{7}}{4}\cdot\frac{5}{8}$

$=\frac{3\sqrt{39}-5\sqrt{7}}{32}$



c. tan(α−β) $=\frac{\sqrt{273}+15}{3\sqrt{39}-5\sqrt{7}}$