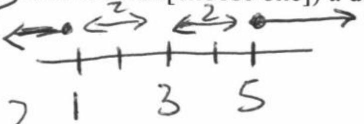
Exam I Practice **MAT 1375** **Spring 2019**

1.(10 pts) Solve each inequality or equality. Write answer in **interval notation** or as **set** if solution set is finite.

a) Use the geometric method with a number line: $|x-3| \geq 2$

Begin by translating into sentence: "set of points ([✓ at least]/no more than[choose one]) a distance 2 from 3 "



$(-\infty, 1] \cup [5, \infty)$

b) Use any method: $|3x-4| = -2$

LHS ≥ 0, RHS < 0 so $\{\}$

2. (25 pts) Given $f(x) = \sqrt{x^2-4}$

a) Find the domain of f.

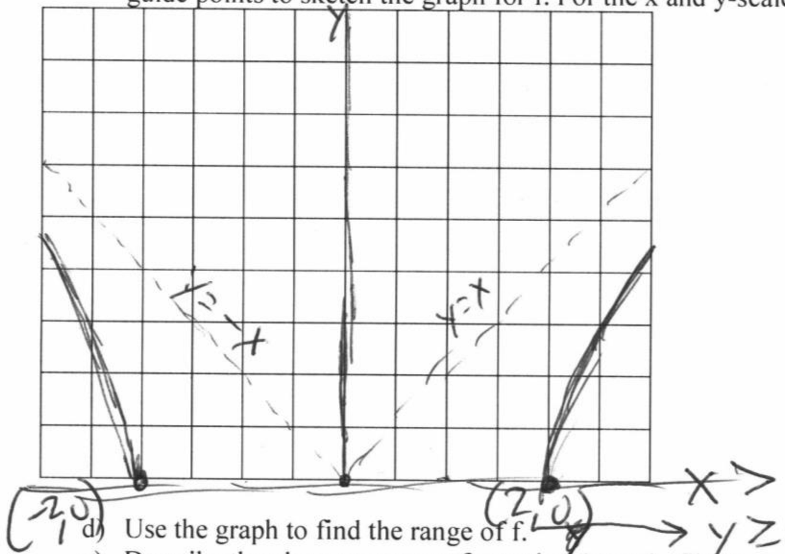
$|x-0| \geq 2$

$x^2 - 4 \geq 0$ $x^2 \geq 4$ $\sqrt{x^2} \geq \sqrt{4}$ $|x| \geq 2$ $(-\infty,-2] \cup [2,\infty)$

b) What happens when x gets large both positively and negatively (use your calculator if need be)? This behavior is related to the existence of asymptotes $y = \pm x$.

as $|x|$ gets large $y \approx |x|$

c) Start by sketching as dashed lines the asymptotes. Find and label 2 guide points. Use the asymptotes and guide points to sketch the graph for f. For the x and y-scales use 1 box is 0.5

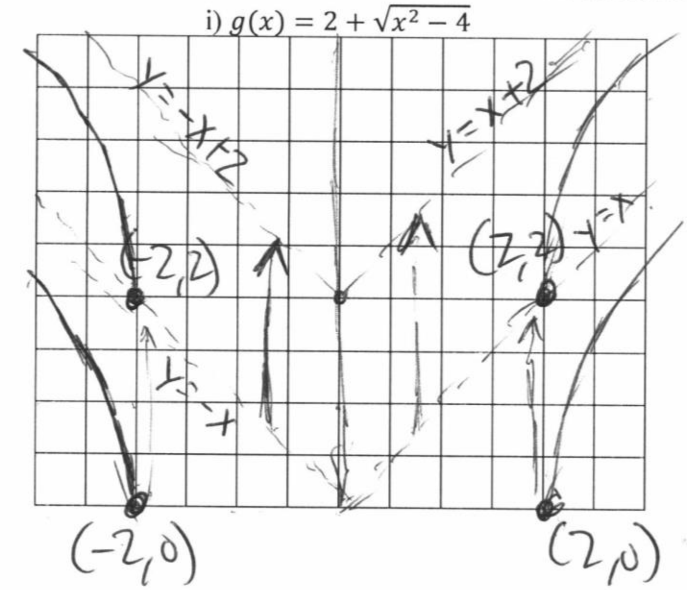


d) Use the graph to find the range of f. → $y \geq 0$ or $[0, \infty)$

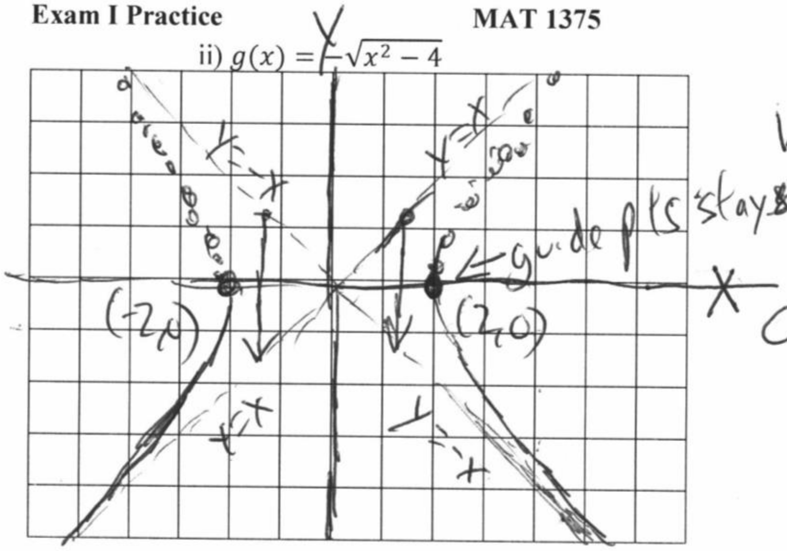
e) Describe the elementary transformation (stretch, flip, translation) that transforms $f(x)$ to $g(x)$. Be as specific/descriptive as possible! Make a graph of f and g on the same set of axes. For the x and y-scales use 1 box is 0.5. Use arrows to show the transformations applied to the guide points and aysmptotes.

i) $g(x) = 2 + \sqrt{x^2-4}$

vertical shift up by 2

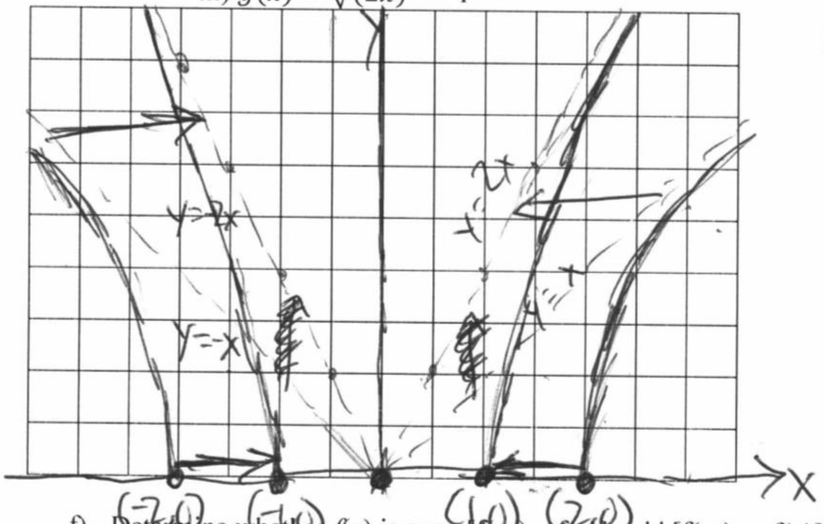


ii) $g(x) = -\sqrt{x^2 - 4}$



vertical flip over
y axis
change scale to
1 box = 1

iii) $g(x) = \sqrt{(2x)^2 - 4}$

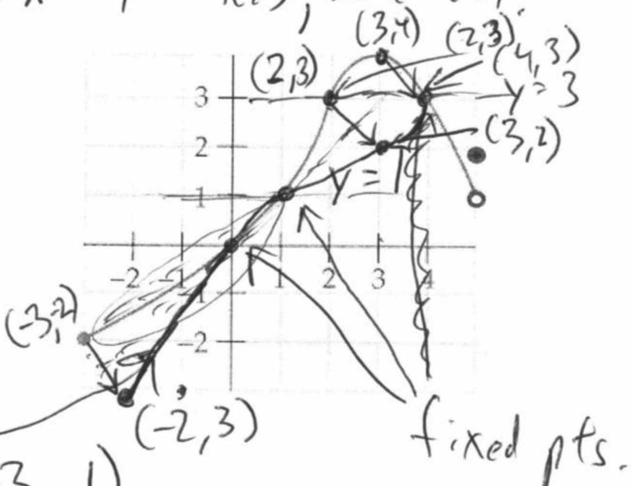


horizontal shrink
by factor of $\frac{1}{2}$

f) Determine whether $f(x)$ is even [$f(-x) = f(x)$], odd [$f(-x) = -f(x)$] or neither. SHOW your work.

$f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4} = f(x)$, so even

3. (20 pts) $g(x)$ is given by the graph below. Find



a) Domain $[-3, 5]$

b) Range $[-2, 4]$

c) g(g(3)) (show work)

$= g(4) = 3$

d) set of x-value(s) where g(x)= 3

$\{2, 4\}$

e) set of x-value(s) (in interval notation) where g(x) < 1.

$[-3, 1)$

g(x) does not have inverse, e.g. fails hor. line test ~~a~~ for y = 3

Max subset [-3, 3]

f) does $g(x)$ have an inverse? Why or why not? If not, choose a maximal subset of the domain working from the left so that it does have an inverse and graph the inverse on the same set of axes on previous page. Be sure to sketch as a dashed line y = x and indicate any fixed points. Select and label 3 nonfixed points of g. Show explicitly using arrows the images of these 3 nonfixed points and label their coordinates.

4. (25 pts) a) Find a root of $p(x) = x^3 + 6x^2 + 10x + 4$ by using calculator or by guessing and checking.

$p(-1) = -1 + 6 - 10 + 4 \neq 0$ $p(-2) = -8 + 24 - 20 + 4 = 0$ so $x = -2$

b) Use factor associated with root from part a) & polynomial division to find a factor of degree 2 for $p(x)$.

$$\begin{array}{r} x^2 + 4x + 2 \\ x + 2 \overline{) x^3 + 6x^2 + 10x + 4} \\ \ominus\, x^3 \overset{\ominus}{+} 2x^2 \\ \hline 4x^2 + 10x \\ \ominus\, 4x^2 + \overset{\ominus}{8}x \\ 2x + 4 \\ \ominus\, 2x + 4 \\ \hline 0 \end{array}$$

$x^2 + 4x + 2$

c) Find the other 2 roots exactly and use your calculator to find approximations to the nearest 10th.

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(2)}}{2} = -2 \pm \sqrt{2} \approx -0.6$$ -3.4

d) Using the results of parts a) and c), write the down the 3 x-intercepts as a set of 3 coordinates.

{(-2, 0), (-3.4, 0), (-0.6, 0)}

e) Find the y-intercept (as a coordinate).

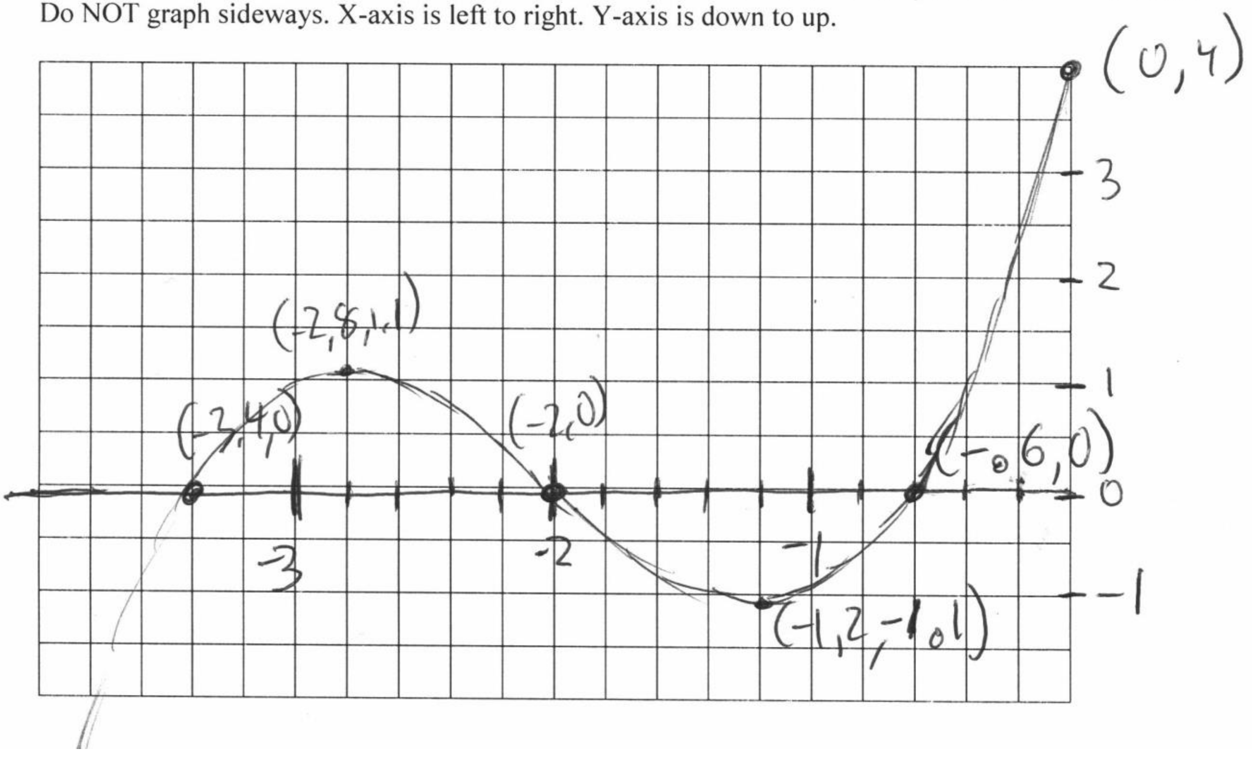
(0, 4)

f) Use your calculator to find the coordinates of the local extrema to the nearest 10th.

Max: (-2.8, 1.1) Min: (-1.2, -1.1)

g) On the grid below, sketch graph. Be sure to mark the x and y-intercepts and the local extrema and label them with their coordinates. For the x-scale use 1 box is 0.2. For the y-scales use 1 box is 0.5.

Do NOT graph sideways. X-axis is left to right. Y-axis is down to up.



5. (20 pts) a) Use algebra to find the inverse function of $f(x)=\dfrac{2x-3}{2x+1}$ and call it g.

$$y=\frac{2x-3}{2x+1} \quad \Big| \quad x=\frac{2y-3}{2y+1}$$

$$2xy+x=2y-3$$

$$2xy-2y=-x-3$$

$$y(2x-2)=-x-3$$

$$y=\frac{-x-3}{2x-2}$$

$$g(x)=f^{-1}(x)=\frac{-x-3}{2x-2}$$

b) Compose the new function g with f as a check (for full credit, ok to just do one of the two compositions).

$$g\circ f(x)=g(f(x))=\frac{-\left(\frac{2x-3}{2x+1}\right)-3}{2\left(\frac{2x-3}{2x+1}\right)-2}\cdot\frac{\frac{2x+1}{1}}{\frac{2x+1}{1}}$$

$$\boxed{\frac{-2x+3-6x-3}{4x-6-4x-2}=\frac{-8x}{-8}=x}$$

so checks

c) What is the domain for f?

$\mathbb{R}\setminus\{-\frac{1}{2}\}$ or $(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},\infty)$

d) What is range of f? Hint: find horizontal asymptote by thinking about what happens when x gets large (both postively and negatively).

$\mathbb{R}\setminus\{1\}$ or $(-\infty,1)\cup(1,\infty)$.

e) Use information from c & d, together with your knowledge about inverse functions to fill in the chart:

Function	**Domain**	**Range**
f	$(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},\infty)$	$(-\infty,1)\cup(1,\infty)$
g	$(-\infty,1)\cup(1,\infty)$	$(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},\infty)$