MAT 1372 Stat w/ Prob classwk 20 fall 2016

Sections 6.5 to 6.7 are somewhat antiquated as they focus on tables for many of the calculations. Nowadays it makes more sense to use a calculator or software such as Excel. However, you must do the problems by converting to and from the standard situation. The pedagogical reason is that if we use Excel functions like NORMDIST, then there is not really much calculation to do and you miss out on the valuable grappling with the material that happens by forcing you to use the standard versions of the Excel functions, such as NORMSDIST. Also, you must draw pictures for each problem. Again the reason is to help solidify the topic in your minds.

* Section 6.5: we convert to Standard and then use **NORMSDIST** to find probabilities
* Section 6.6: some important **properties**
* Section 6.7: given probabilities, we use **NORMSINV** to find “zscores”, then “Un”standardize to find scores.

**6.5 FINDING NORMAL PROBABILITIES (via STANDARD NORMAL and NORMSDIST)**

Given an inequality for a normal random variable, we “standardize” by subtracting the mean and dividing by the standard deviation

**Example**: Suppose that μ=70 and σ=10, what chance did someone have of getting a 90? An 85?

Since 90 is 2 standard deviations from the mean, using the graph, we see that the chance of getting a 90 is .5-(.34+.135)=.025=2.5%

85 is 1.5 std devs away from the mean, so the chart only tells us that is between 2.5% and 16%. More formally, we standardize:



To get the probability, we think complement



And use the Excel function 1-NORMSDIST(1.5) to get 0.066807 or about 6.7%

**2.** If *X* is normal with mean 10 and standard deviation 3, find

**(a)** *P*{*X* > 12} **(c)** *P*{8 < *X* < 11} **(e)** *P*{|*X* − 10| > 5}

**6.** The life of a certain automobile tire is normally distributed with mean

35,000 miles and standard deviation 5000 miles.

1. What proportion of such tires last between 30,000 and 40,000 miles?

*P*{|*Z* | < 1}

1. What proportion of such tires last over 40,000 miles?

*P*{*Z*  >1}

1. What proportion last over 50,000 miles?

*P*{*Z*  >3}

**7.** Suppose you purchased such a tire as described in Prob. 6. If the

tire is in working condition after 40,000 miles, what is the conditional

probability that it will still be working after an additional 10,000 miles?

 *P*{*Z*  >3}/ *P*{*Z*  >1}

**6.6 PROPERTIES OF NORMAL RANDOM VARIABLES**

Recall:

* for random vars X and Y, *E*[*X* + *Y*] = *E*[*X*] + *E*[*Y*] = μ*x* + μ*y*
* for **INDEPENDENT** random vars X and Y:

Var(*X* + *Y*) = Var(*X*) + Var(*Y*) = 

Suppose *X* and *Y* are independent **normal** random variables with means μ*x* and

μ*y* and standard deviations σ*x* and σ*y* , respectively. Then *X* + *Y* has

mean: *E*[*X* + *Y*] = μ*x* + μ*y*

standard deviation: SD(*X* + *Y*) = 

and is **normal** (needs proof but we just accept as fact).

**6.7 PERCENTILES OF NORMAL RANDOM VARIABLES**

**(the inverse problem)**

The **zscore** *z*α is the value for which *P*{*Z* > *z*α} = α:

 The zscore is closely related to the percentile.

For example, if on a test, 75% of the scores were below 30, then 30 is the 75th percentile and 30 is the zscore for α=.25.

Think of a zscore as a **function** which inputs a probability 0<α<1 and outputs a real number.

**1.** Find to two decimal places:

**(a)** *z*0.07 =Normsinv(1­­­­­­­­­­­­­−0.07)

**(e)** *z*0.65=Normsinv(1­­­­­­­­­­­­­−0.65)

**(f)** *z*0.50

**(g)** *z*0.95

**2.** Find the value of *z* for which

**(a)** *P*{|*Z*| > *z*} = 0.05

 z=Normsinv(1−0.025)

Recall that. If we solve for x we get



Once a zscore is found, we use the above formula to “de”standardize.

**3.** If *X* is a normal random variable with μ=50 and σ=6,

find the approximate value of **x**for which

**(d)** *P*{*X* < **x**} = 0.05 **x** =6\*Normsinv(0.05)+50

**(e)** *P*{*X* < **x** } = 0.88

 **(a)** *P*{*X* > **x** } = 0.5

**(b)** *P*{*X* > **x** } = 0.10 **x** =6\*Normsinv(1­−0.1)+50

**(c)** *P*{*X* > **x** } = 0.025

(f) *P*{|*X*−50| > **x** } = 0.05

(g) *P*{|*X*−50| < **x** } = 0.65

**6.** The time it takes for junior high girls to run 1 mile is normally distributed

with mean 460 seconds and standard deviation 40 seconds.

As a selection mechanism, the track team will only take girls that run in the top 20%. What is the critical time below which the girl must reach to make the team?

*P*{*X* < **x**} = 0.2 **x** =40\*Normsinv(0.2)+460=426 seconds or 7 min 6 sec.

**9.** The amount of radiation that can be absorbed by an individual before

death ensues varies from individual to individual. However, over the

entire population this amount is normally distributed with mean 500

roentgens and standard deviation 150 roentgens. Above what dosage

level will only 5 percent of those exposed survive?

Answer: 747

Apparently, these numbers were gathered from the Chernobyl accident. The roentgen is also known as rem. Since then, the unit used for radiation absorption has been changed to the sievert (1sievert = 100 rems). See a recent [NYTimes graph](http://www.nytimes.com/interactive/2011/03/26/world/asia/dangers-of-radiation-for-workers-at-fukushima-daiichi.html?ref=asia)  for a comparison of the exposure that the Fukushima workers were receiving with that from Chernobyl:

1. **Dangers of Radiation for Workers at Fukushima Daiichi**

Inside the buildings of the damaged reactors at Fukushima Daiichi, workers attempting to make repairs are facing dangerous risks from radiation exposure. The burns suffered by two workers on Friday are just one indication of the perilous levels that likely exist in a number of areas in the plant. In these conditions, rapid exposure — in minutes to a few hours — to lethal doses is possible.

