MAT 1372 Stat w/ Prob classwk 17 Fall 2016

**5.5 BINOMIAL RANDOM VARIABLES**

In this section we consider a repetition of the same trial with only 2 outcomes:

Failure or 0

Success or 1

The **probability of success is p** (and hence that of failure 1-p).

For example,

* a fair coin, p=.5
* a die and consider success only if a 1 is rolled, p=1/6

Let’s look at the case of the coin toss for only 2 trials:

|  |  |  |  |
| --- | --- | --- | --- |
| result  of 2 tosses | outcome of random variable | probability calculation | probability |
| TT | 0 | (.5)^2 | .25 |
| TH, HT | 1 | 2\*(.5)^2 | .5 |
| HH | 2 | (.5)^2 | .25 |

Staying with 2 trials but using a **general p**, we get:

|  |  |  |  |
| --- | --- | --- | --- |
| outcomes | 0 | 1 | 2 |
| probability | (1-p)^2 | 2(p)(1-p) | (p)^2 |

Now, let’s look at 3 trials:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| outcomes | 0 | 1 | 2 | 3 |
| probability | (1-p)^3 | 3(p)(1-p)^2 | 3 (p)^2(1-p) | (p)^3 |

In general, we get for n trials:



In Excel:

BINOMDIST(i,n,p,false)= 

BINOMDIST(i,n,p,true)= 

For n=10 and p=.5, .3 & .6 the distribution is calculated and illustrated in the accompanying spreadsheet.

Below are 3 graphs from the textbook illustrating the effect of changing p.



**5.5.1 Expected Value and Variance of a Binomial Random**

**Variable**

Exercise: a) Calculate expected value and variance of one trial. To find Variance, use

b) Use the Expectation property E[X+Y]=E[X]+E[Y] valid for the sum of any random variables iterated n times to show that

**expectation** for **n trials** is **np**.

c) Similarly, use the Variance property V[X+Y]=V[X]+V[Y] valid only for the sum of independent random variables iterated n times to show that

**variance** for **n trials** is **np(1-p)**.

**6.** Each ball bearing produced is independently defective with probability

0.05. If a sample of 5 is inspected, find the probability that

**(a)** None are defective.

**(b)** Two or more are defective.

**7.** Suppose you will be attending 6 hockey games. If each game independently

will go to overtime with probability 0.10, find the probability that

**(a)** At least 1 of the games will go into overtime.

**(b)** At most 1 of the games will go into overtime.

**8.** A satellite system consists of 4 components and can function if at

least 2 of them are working. If each component independently works

with probability 0.8, what is the probability the system will function?

**9.** A communications channel transmits the digits 0 and 1. Because of

static, each digit transmitted is independently incorrectly received

with probability 0.1. Suppose an important single-digit message is

to be transmitted. To reduce the chance of error, the string of digits

0 0 0 0 0 is to be transmitted if the message is 0 and the string 1 1 1 1 1

is to be transmitted if the message is 1. The receiver of the message

uses “majority rule” to decode; that is, she decodes the message as 0

if there are at least 3 zeros in the message received and as 1 otherwise.

**(a)** For the message to be incorrectly decoded, how many of the

5 digits received would have to be incorrect?

**(b)** What is the probability that the message is incorrectly decoded?

**13.** Four fair dice are to be rolled. Find the probability that

**(a)** 6 appears at least once.

**(b)** 6 appears exactly once.

**(c)** 6 appears at least twice.

**16.** Let *X* be a binomial random variable with parameters *n* = 20 and

*p* = 0.6. Find

**(a)** *P*{*X* ≤ 14} **(b)** *P*{*X* < 10} **(c)** *P*{*X* ≥ 13}

**(d)** *P*{*X* > 10} **(e)** *P*{9 ≤ *X* ≤ 16} **(f)** *P*{7 < *X* < 15}

**17.** A fair die is to be rolled 20 times. Find the expected value of the number

of times

**(a)** 6 appears.

**(b)** 5 or 6 appears.

**(c)** An even number appears.

**(d)** Anything else but 6 appears.

**19.** The probability that a fluorescent bulb burns for at least 500 hours is

0.90. Of 8 such bulbs, find the probability that

**(a)** All 8 burn for at least 500 hours.

**(b)** Exactly 7 burn for at least 500 hours.

**(c)** What is the expected value of the number of bulbs that burn for at

least 500 hours?

**(d)** What is the variance of the number of bulbs that burn for at least

500 hours?

**20.** If a fair coin is flipped 500 times, what is the standard deviation of the

number of times that a head appears?

**22.** The expected number of heads in a series of 10 flips of a coin is 6. What

is the probability there are 8 heads?

**24.** If *X* is a binomial random variable with expected value 4.5 and variance

0.45, find

**(a)** *P*{*X* = 3} **(b)** *P*{*X* ≥ 4}