MAT 1372 Stat w/ Prob classwk 16 fall 16

Review (of 5.3):

The **expected value** of a random variable X (also known as the **mean)** is the sum of the outcomes weighted by their probabilities:



**Properties of Expected Values**

If *c* is a constant, then:

*E*[*cX*] = *cE*[*X*]

*E*[*X* + *c*] = *E*[*X*] + *c*

Exercise: Put these properties into words.

For any random variables *X* and *Y*,

\*\**E*[*X* + *Y*] = *E*[*X*] + *E*[*Y*]

**Exercise:**

Suppose that 2 batteries are randomly selected from a drawer containing

8 good and 2 defective batteries. Let *W* denote the number of defective batteries selected.

**(a)** Find *E*[*W*] by first determining the probability distribution of *W*.

Let *X* equal 1 if the first battery chosen is defective, and let *X* equal

0 otherwise. Let *Y* equal 1 if the second battery is defective and

equal 0 otherwise.

**(b)** Give an equation relating *X*, *Y* and *W*.

**(c)** Use the equation in (b) to obtain *E*[*W*].

We can extend property to expectation of sum of 3 random variables *X, Y* and Z: \*\*\**E*[*X* + *Y + Z*] = *E*[*X*] + *E*[*Y*] + *E*[*Z*], of sum of 4 variables W + *X* + *Y + Z,* etc.

**Exercise:**

1. Find the expectation for flipping a fair coin if the outcomes are 0 if tails and 1 if head.
2. Without actually finding distributions, use a. and properties \*\*, \*\*\*, etc. to find

|  |  |
| --- | --- |
| **flip a coin** | **expected number of heads** |
| twice | *E*[*X* + *Y*] = *E*[*X*] + *E*[*Y*] = |
| 3 times | *E*[*X* + *Y + Z*] = *E*[*X*] + *E*[*Y*] + *E*[*Z*]= |
| 10 times | E[X1 + X2 +…+ X10] = E[X1] + E[X2 ] +…+ E[X10]= |
| *n* times | E[X1 + X2 +…+ Xn] = E[X1] + E[X2 ] +…+ E[Xn]= |

**5.4 VARIANCE OF RANDOM VARIABLES**

We expect a random variable *X* to take on values around its mean *E*[*X*].

We might measure the **spread** of *X* by seeing how far (on average) *X* is from its mean, i.e.,

*E*[|*X* − μ|].

However, it turns out to be more convenient to consider not the absolute value but the square of the difference from the mean:



As before, the standard deviation is the square root of the variance:



**Exercise, use the properties of expectation to show**:



**Example**

Find Var(*X*) for the random variable *X* of a fair coin flip:

|  |  |  |
| --- | --- | --- |
| *x* | *1* | *0* |
| *P(X=x)* | *.5* | *.5* |

**Solution**

1. Find the mean (expectation):

1\* .5+0\*.5=.5

1. Find the square of the deviation (difference) of each data value from mean:

|  |  |  |
| --- | --- | --- |
| *(x-)^2* | *(1-.5)^2=.25* | *(0-.5)^2=.25* |
| *P(X=x)* | *.5* | *.5* |

1. Multiply by the respective probabilities and sum to get variance.

.25\*.5+.25\*.5=.125+.125=.25

1. Take square root to get SD

SD= Sqrt(.25)=.5

Should not be surprise: on average (in fact always) outcomes are ½ unit from mean.

**Example 5.12 (generalize to a bent or unfair coin)**

Find Var(*X*) when the random variable *X* is defined by:

|  |  |  |
| --- | --- | --- |
| *x* | *1* | *0* |
| *P(X=x)* | *p* | *1-p* |

**Solution**

1. Find the mean (expectation):

1\* p+0(1-p)=p

1. Find the square of the deviation (difference) of each data value from mean:

|  |  |  |
| --- | --- | --- |
| *(x-)^2* | *(1-p)^2* | *(0-p)^2* |
| *P(X=x)* | *p* | *1-p* |

1. Multiply by the respective probabilities and sum to get variance.



1. Take square root to get SD



In the Excel file, these expressions are calculated for various values of p.

For another example, we return to the bakery and cake problem (see excel file).

We conclude that the variance for the number of customers asking for cake is 1.8.

And that for the number of cakes unsold is 1.06.

**General Properties of Variance and SD**







**If X and Y are independent:**



**Exercise:**

1. Prove the 2 SD properties from the definition of SD and the properties of Var

2. If X and Y are independent, express SD(X+Y) in terms of SD(X) and SD(Y)

(Hint: think Pythagorean Theorem.)

**13v.** A lawyer charges a fixed fee of $2000 or

takes a contingency fee of $8000 if she wins the case (and $0 if she

loses). She estimates that her probability of winning is 0.3.

Determine the expectation and standard deviation of her fee if

**(a)** She takes the fixed fee.

**(b)** She takes the contingency fee.

**(c)** Do you think the lawyer’s fee structure is fair? Explain.

**17v.** If SD(*X*) = 4, what is Var(3*X*)?

**18v.** If SD(3*X* + 2) = 9, what is Var(*X*)?

**19.** If *X* and *Y* are independent random variables, both having variance 1, find

**(a)** Var(*X* + *Y*)

**(b)** Var(*X* − *Y*)