New York City College of Technology MAT1372 Practice_Exam II solns, Halleck, Fall 2013

Show all work and answers in blue book. You may use scientific or graphing calculator. Pts for problems sum to 90. For remaining 10 pts, submit handwritten 2-sided **quality** formula sheet of all the material needed for the exam. When finished, make sure name is on everything which you submit; **without folding,** insert formula sheet, exam sheet and graph paper into blue book.

1. (10 pts) A random variable has the following probability distribution:

X	-2	-1	1	2
P(X)	0.3	0.2	0.3	?

Find and interpret a) the probability when X=2: **P(2)=1-(.3+.2+.3)=.2 The probability of 2 as an outcome is .2 or 20%.**

b) the mean: **E(X)= -2*.3+-1*.2+1*.3+2*.2=-0.1 On average an outcome will be -0.1.**

c) the standard deviation **V(X)= E(X^2)-mu^2=(1.2+.2+.3+.8)-(-.1)^2=2.5-.01=2.49 so SD=√2.59=1.58**

x^2	4	1	1	4
P(x)	0.3	0.2	0.3	0.2
x^2*P(x)	1.2	0.2	0.3	0.8

On average an outcome will be about 1.6 units from the mean of -0.1.

2. (10 pts) If a bent coin with probability of a head .3 is tossed 5 times, find the probability of getting
a) exactly 0 heads **(5c0)(.3)^0(.7)^5=.168** b) exactly 1 head **(5c1)(.3)^1(.7)^4=.360**

c) at least 2 heads [use results from parts a) and b)]**=1-(.168+.360)=.472**

d) write down an excel command that you could use directly to answer c) without using answers from a and b
=1-BINOMDIST(1,5,0.3,TRUE)

3. **(**10 pts) An insurance company estimates that 5% of all its customer's insurance claims are fraudulent. The company received 57,000 claims last year. Let X_i be random variable which is 1 if i^{th} complaint is fraudulent and 0 otherwise. Let *X* be the random variable which counts the number of fraudulent claims last year.

a) Find the expectation for X_i and *X* and interpret.

x_i	0	1
$P(x_i)$	.95	.05

E[Xi]=.05: for a single trial, the expectation is the same as probability of a success.
E[X]=np=57000*.05=2850: the insurance company can expect about 2850 claims.

b) Find the standard deviation for X_i and *X* and interpret.

SD[Xi]=sqrt(.05*.95)=.22: on average, the outcome of a single claim will be .22 from mean of .05.
SD[X]=sqrt(57000*.05*.95)=.22: on average, the actual number of fraudulant claims will be 52 from average value of 2850.

4. (10 pts) A pair of dice is rolled 1000 times. Let X_i be the random variable which consists of 1 if the sum of roll *i* is 10 and 0 otherwise. Let *X* be the random variable which counts the number of 10's in the 1000 rolls.

a) There are 3 rolls which contribute to sum of 10, so chance of a 10 is 3/36=1/12. Find the expectation for X_i and *X* and interpret.

E[Xi]=1/12: for a single trial, the expectation is the same as probability of a success.

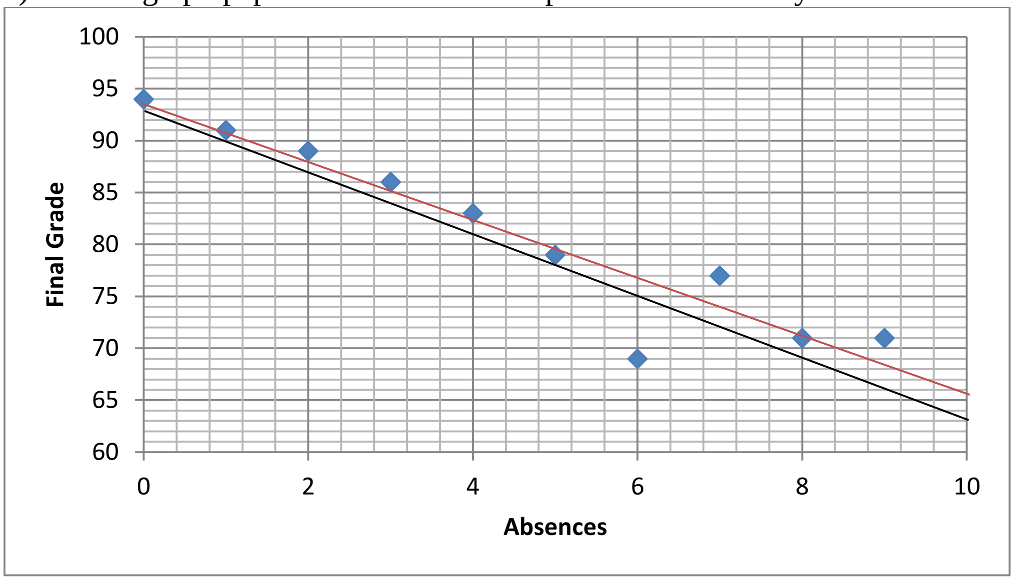
x_i	0	1
$P(x_i)$	11/12	1/12

E[X]=np=1000*1/12=250/3=83.3: on average, there will be 83 rolls that sum to 10.

b) Find the standard deviation for X_i and X and interpret.

So $E[X_i\text{^}2]=1/12$, $V[X_i]=$ **$E(X_i\text{^}2)$-mu^2=1/12-(1/12)^2=1/12(1-1/12)=1/12*11/12=11/144=pq and**
SD[X_i]=sqrt(11)/12=.28: on average, the outcome of a single roll will be .28 from mean of 1/12.
SD[X]= sqrt(1000*1/12*11/12)=5sqrt(110)/6=8.74: on average, the actual number of rolls will be about 9 from average value of 83.

5. (20 pts) The following data represent the number of days absent (X) and final grade (Y) in a statistics course.
a) Use the graph paper to draw the scatter plot. Draw in what you think is the trend line (**in black**).



b) Find and interpret the coefficient of correlation.
r=sum dev prods/sqrt(sum of xdev^2* sum of ydev^2)= -231/sqrt(82.5*726)=-.944
Since r is negative, the correlation is negative (trendline goes down from left to right).

c) **Since r^2 is .891, at most 89% of the response variable may be attributed to the explanatory variable.**

d) Find the equation of the regression line and plot on your scatter plot (**in red**).
B= sum dev prods/sum of xdev^2=-231/82.5=-2.8
A=$\bar{y}$-B$\bar{x}$= 81-(-2.8)4.5=93.6
y=A+Bx=93.6-2.8x
(0, 93.6) and (10, 65.6) were used to plot the line. Please label these points on your graph.

e) Interpret the slope & y-intercept.
For every absence, the final grade decreases by 2.8 points.
A person with 0 absences should have a score of 93.6.

f) If Carol has 5 absences, what final grade can she expect?
y=93.6-2.8*5=79.6 Carol can expect a grade of about 80.

6.(10 pts) If the a probability distribution is in the form of a right triangle with peak at 2 and x-intercept 8, find
a) height of the triangle: **A=1/2bh=1/2(8-0)h=4h=1 so h=1/4**

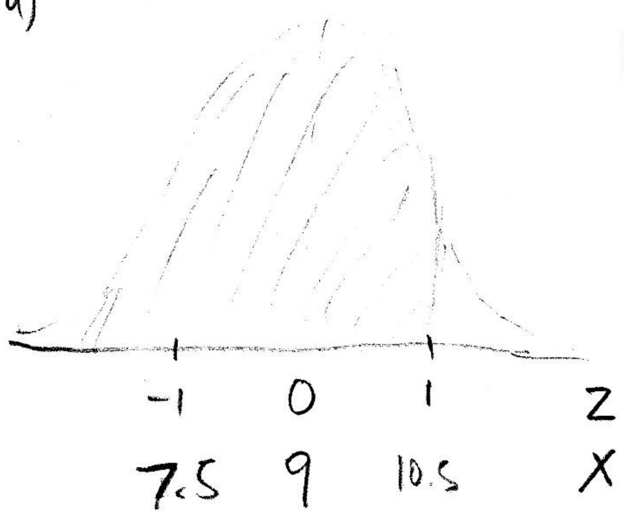
b) P(x<2) **A=1/2bh=1/2(2-0)(1/4)=1/4**

c) P(4<x<6) **Line goes thru points (2,1/4) and (8,0), so equation is y=-1/24(x-8) or y=(8-x)/24, b_1=y(4)=1/6 and b_2=y(6)=1/12. For the trapezoid A=1/2(b_1+ b_2)h=1/2(1/6+1/12)(6-4)=2/12+1/12=3/12=1/4**

7. (20 pts) Men shoe sizes in one ethnic population are normally distributed with mean 9 and standard deviation 1.5. For each part, draw a picture with the normal distribution with both X and Z labels.

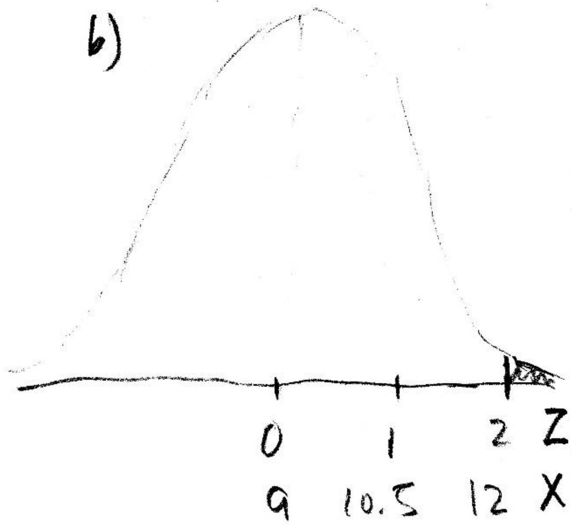
a) Find P(X < 10.5)
b) Find P(X > 12)
c) Write the Excel command to find P(|X−9| >2)
d) Using normsinv, write Excel command to find value of x above which 17% of men have higher shoe sizes

a)



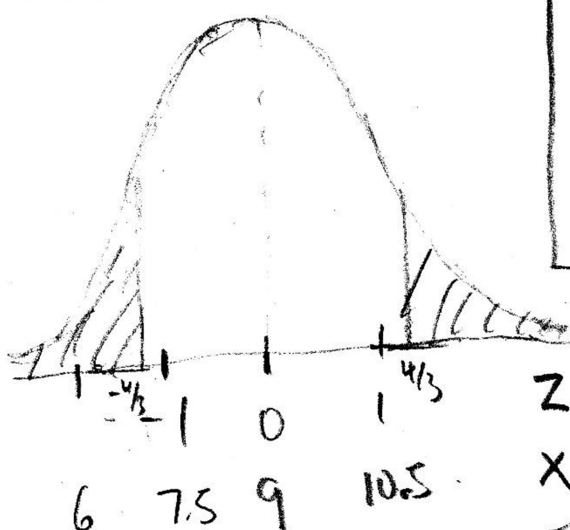
$P(x < 10.5) = P(Z < 1)$
$= .5 + .34 = .84$

b)



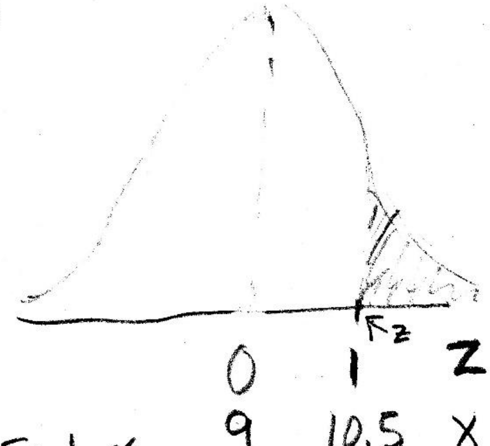
$P(x > 12)$
$= p(z > 2)$
$= .025$

c)



$P(|X - 9| > 2)$
$= P(|Z| > \frac{2}{1.5} = \frac{4}{3})$
$= 2(1 - \text{normsdist}(4/3))$

d)



Find x
so that $P(X > x) \geq .17$
$z = \text{normsinv}(1 - .17)$
$x = 1.5 \text{ norms inv}(1 - .17)$
$+ 9$