Practice Exam II v2 solutions Halleck MAT 1275 Spring 2017

Exam will last for exactly 1 hour. (The other 40 minutes will be devoted to the new material as scheduled.)

1. (5 points) Simplify each radical, complete the multiplication and then simplify once more.

$$5\sqrt{-3x}\left(2\sqrt{28x^2}+4x\sqrt{-39x^5}\right)$$
$$=5i\sqrt{3x}\left(4x\sqrt{7}+4x^3i\sqrt{39x}\right)$$
$$=20xi\sqrt{21x}+20x^3i^2\sqrt{3\cdot 3\cdot 13x^2}$$
$$=20xi\sqrt{21x}-60x^4\sqrt{13}$$
$$=-60x^4\sqrt{13}+20xi\sqrt{21x}$$

2. (5 points) Divide, leave in standard complex form:

$$\frac{6-4i}{10+9i}\cdot\frac{10-9i}{10-9i}=\frac{60-54i-40i+36i^2}{100-81i^2}=\frac{24-94i}{181}=\frac{24}{181}-\frac{94}{181}i$$

3. (10 points) Put into standard form $ax^2+bx+c=0$ and then solve using completing the square:

$(x+3)^2+x^2+3=14x \Rightarrow 2x^2-8x+12=0$ To complete the square, we begin by dividing by 2:

$$x^2-4x+6=0 \Rightarrow x^2-4x+\_\_\_=-6+\_\_\_ \Rightarrow x^2-4x+\_\underline{4}\_=-6+\_\underline{4}\_$$
$$\Rightarrow (x-2)^2=-2 \Rightarrow x-2=\pm i\sqrt{2} \Rightarrow x=2\pm i\sqrt{2}$$

4. (10 points) Put into standard form $ax^2+bx+c=0$ and then solve using the quadratic formula:

$$3x^2+11=14x \Rightarrow 3x^2-14x+11=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-14)\pm\sqrt{(-14)^2-4\cdot 3\cdot 11}}{2\cdot 3}$$
$$=\frac{14\pm\sqrt{196-132}}{6}=\frac{14\pm\sqrt{64}}{6}=\frac{14\pm 8}{6}\quad\left\{1,\frac{22}{6}=\frac{11}{3}\right\}$$

5. (15 points) Find the vertex and and y-intercept. Use factoring to solve the quadratic equation to find the x-intercepts. **Mark these 4 points on the provided graph paper and label with their coordinates.** Use the 4 points to help you sketch the quadratic function (parabola).

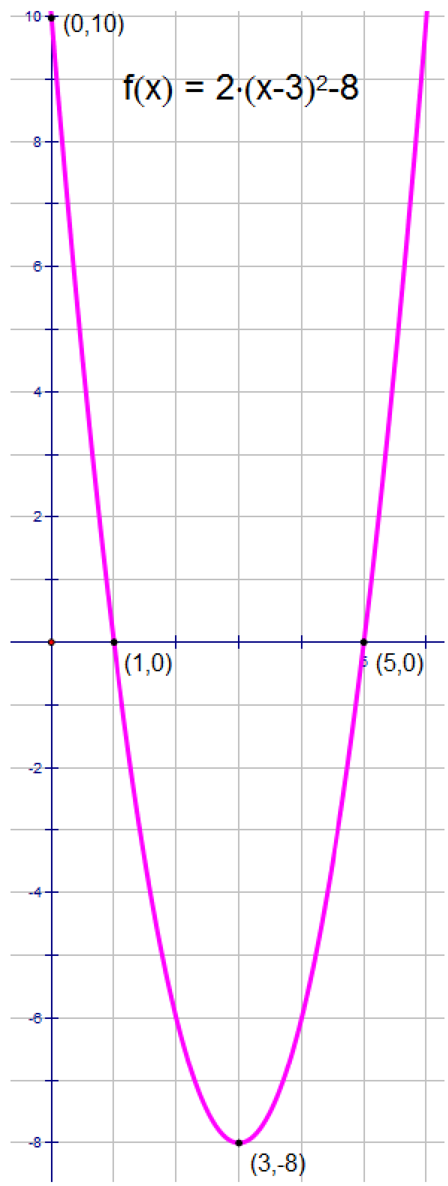
$y=2x^2-12x+10=2(x^2-6x+5)$ so y-int is $(0,10)$

$$\Rightarrow\frac{y}{2}=x^2-6x+5\;\;\frac{y}{2}-5+\_\_\_=x^2-6x+\_\_\_$$
$$\Rightarrow\frac{y}{2}-5+\_\underline{9}\_=x^2-6x+\_\underline{9}\_$$
$$\Rightarrow\frac{y}{2}+4=(x-3)^2\Rightarrow y=2(x-3)^2-8$$

so vertex is $(3,-8)$

$y=0\Rightarrow 2(x-1)(x-5)=0\Rightarrow x=1$ or 5

so x-intercepts are $(1,0)$ and $(5,0)$



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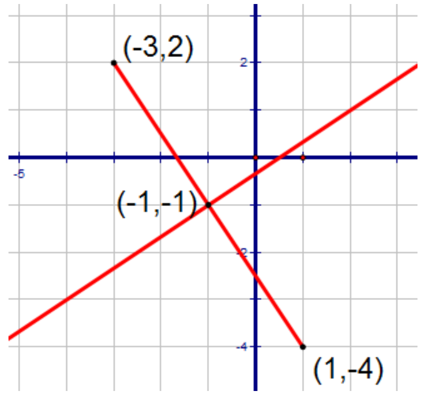
6. (10 points) If the endpoints of a line segment are **(−3, 2)** and **(1, −4)**. Find the equation of the perpendicular bisector in slope intercept form. Check by providing a quick sketch below of the line segment and the line.

The midpoint is (-1,-1).
The slope of the line segment is
(-4-2)/(1-(-3))=-6/4=-3/2.
Therefore, the slope of the perpendicular line is 2/3.
Hence the equation for the ⊥ bisector is
y+1=2/3(x+1) or y=(2/3)x-1/3



7. (15 pts) Put equation of circle into standard form by completing square for both x and y. On **graph paper provided**, mark center as well as 4 points on circle. **Label 5 points with their coordinates.** Sketch graph.

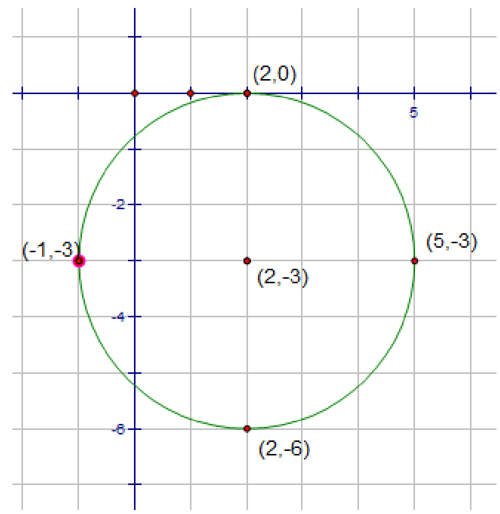
$x^2+y^2-4x+6y+4=0$

$x^2-4x+\_\_\_+y^2+6y+\_\_\_=-4+\_\_\_+\_\_\_$

$x^2-4x+\_\underline{4}\_+y^2+6y+\_\underline{9}\_=-4+\_\underline{4}\_+\_\underline{9}\_$

$(x-2)^2+(y+3)^2=9$

So center is (2,-3) and radius is 3



8. (15 points) The gas mileage of a car is given by the function $M(x) = -0.012x^2 + 1.2x + 10$ where x represents the speed in miles per hour and M (x) is given in miles per gallon. Equation is valid from x = 40 mph until x = 80 mph. At what speed will the car get the maximum gas mileage?
Find the vertex using formula method. Find the mileage at the lower and upper values of interval. Use these 3 points to provide a quick sketch below. Answer question with a complete sentence (include the units!).

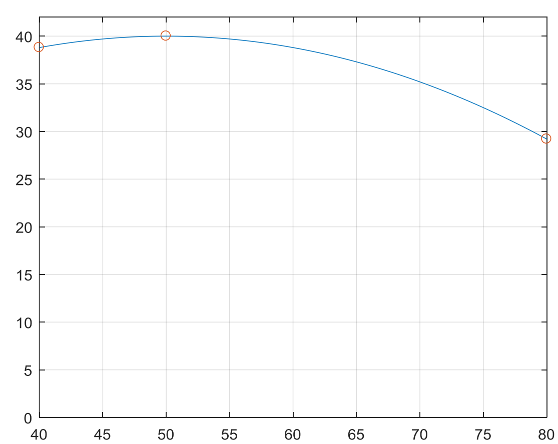
x coordinate of vertex is

$$-\frac{b}{2a}=-\frac{1.2}{2(-.012)}=50$$

Now let's create a table and plot the points

x	**y**
40	38.2
50	40
80	29.2

Vertex is (50,40)
The car will get maximum gas mileage when it is travelling 50 mph.



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9. (15 points) Solve the 3x3 system. You MUST use elimination. (Substitution not allowed.)
Hint: look for variable with 1 or -1 as coefficient and eliminate it.
z has a coefficient of 1 in the first equation so we eliminate it from each of the pairings of equation 1 with the other 2 equations.

$$\begin{cases} 2x-3y+z=-9 \\ 3x+5y+2z=16 \\ -4x+2y-3z=4 \end{cases}$$

$$\begin{cases} 2x-3y+z=-9 \\ 3x+5y+2z=16 \end{cases} \Rightarrow \begin{cases} -4x+6y-2z=18 \\ 3x+5y+2z=16 \end{cases} \Rightarrow -x+11y=34$$

$$\begin{cases} 2x-3y+z=-9 \\ -4x+2y-3z=4 \end{cases} \Rightarrow \begin{cases} 6x-9y+3z=-27 \\ -4x+2y-3z=4 \end{cases} \Rightarrow 2x-7y=-23$$

We now have 2x2 system in the x and z variables.
Again there is a variable with a 1 or -1, namely x, so we eliminate it:

$$\begin{cases} -x+11y=34 \\ 2x-7y=-23 \end{cases} \Rightarrow \begin{cases} -2x+22y=68 \\ 2x-7y=-23 \end{cases}$$

$15y=45$ so $y=3$

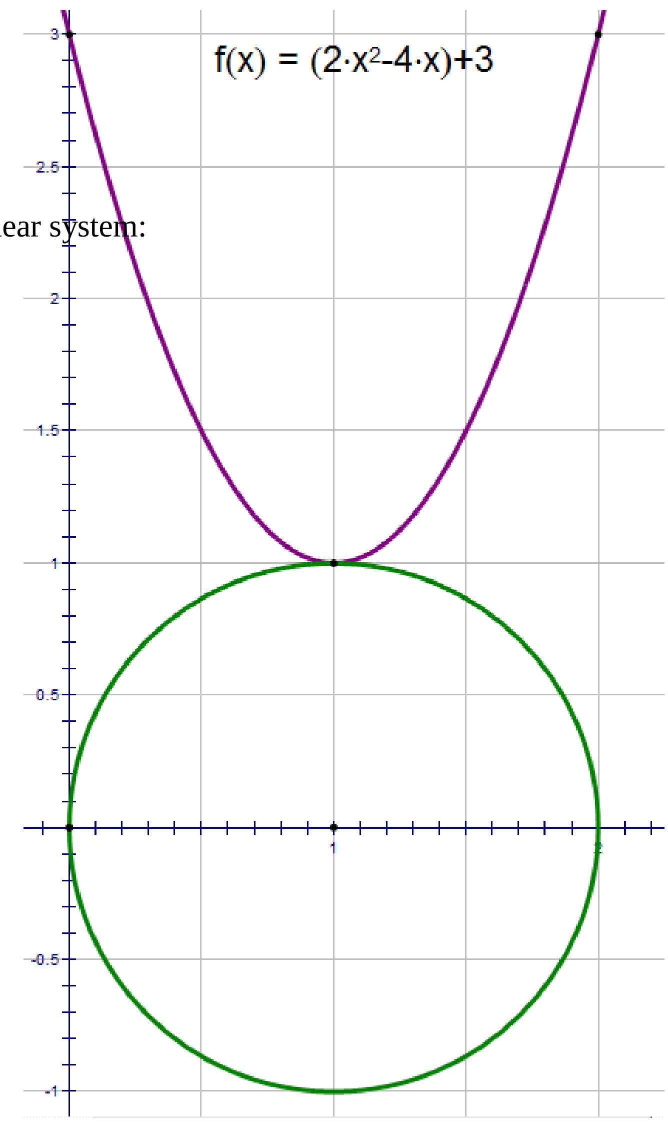
Substitute to get x:

$-x+11(3)=34 \Rightarrow$ so $x=-1$

Now substitute to get z:

$2(-1)-3(3)+z=-9 \Rightarrow z=2$

and the solution is (-1, 3, 2)



Extra credit (10 points) Solve and graph as a check the nonlinear system:

$$\begin{cases} y=2x^2-4x+3 \\ x^2-2x+y^2=0 \end{cases} \Rightarrow \begin{cases} -2x^2+4x+y \qquad =3 \\ 2x^2-4x \qquad +2y^2=0 \end{cases}$$

$2y^2+y=3 \Rightarrow 2y^2+y-3=0$

$(2y+3)(y-1)=0 \Rightarrow y=-\frac{3}{2}$ or 1

$y=-\frac{3}{2}: \quad x^2-2x+\left(-\frac{3}{2}\right)^2=0$ or $4x^2-8x+9=0$

$b^2-4ac=(-8)^2-4(4)9=-80$ so no real solution

$y=1: \quad x^2-2x+1=0$ or $(x-1)^2=0$ so $x=1$

and (1,1) is the only solution. Now we do graph:

$y=2x^2-4x+3 \Rightarrow \frac{y}{2}-2+\_\_\_=x^2-2x+\_\_\_$

$\frac{y}{2}-\frac{3}{2}+\_1\_=x^2-2x+\_1\_ \Rightarrow \frac{y}{2}-\frac{1}{2}=(x-1)^2$

$\Rightarrow y=2(x-1)^2+1$

$x^2-2x+\_\_\_+y^2=0+\_\_\_$

$\Rightarrow x^2-2x+\_1\_+y^2=0+\_1\_$

$\Rightarrow (x-1)^2+y^2=1$