

**Practice Exam III Halleck solutions MAT 1275**

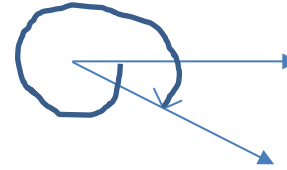
**Spring 2017**

Exam will last for exactly 1 hour. (The other 40 minutes will be devoted to the new material as scheduled.)

1. (10 pts) Convert  $17\pi/6$  radians to degrees and  $-400$  degrees to radians. Also draw each of the angles.

$$\frac{17\pi}{6} \cdot \frac{180 \text{ deg}}{\pi} = \frac{17\cancel{\pi}}{1} \cdot \frac{30 \text{ deg}}{\cancel{\pi}} = 510 \text{ deg}$$

$$\frac{-400 \text{ deg}}{1} \cdot \frac{\pi}{180 \text{ deg}} = \frac{-400 \text{ deg}}{180 \text{ deg}} \cdot \pi = -20/9 \pi$$



2. (16 points) Prove the following identity:

$$\begin{aligned} \frac{\cos x}{2-2\cos x} - \frac{\cos x}{2+2\cos x} &= \cot^2 x \\ \frac{\cos x}{2} \left[ \frac{1}{1-\cos x} - \frac{1}{1+\cos x} \right] &= \frac{\cos^2 x}{\sin^2 x} \\ \frac{\cos x}{2} \left[ \frac{(1+\cos x) - (1-\cos x)}{(1-\cos x)(1+\cos x)} \right] &= \\ \frac{\cos x}{2} \left[ \frac{2\cos x}{1-\cos^2 x} \right] &= \\ \frac{\cos^2 x}{\sin^2 x} &= \end{aligned}$$

3. (20 pts) Solve **exactly** each of the equations for  $[0, 2\pi)$ . NO PICTURE NO CREDIT.

$\cot^2 x = 3$

$\tan^2 x = 1/3$

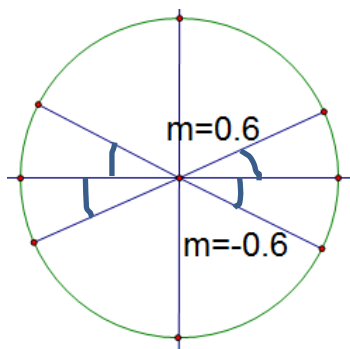
$\tan x = \pm 1/\sqrt{3}$

a)  $x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

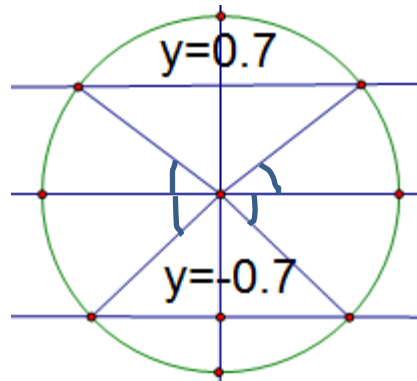
$2\sin^2 x - 1 = 0$

$\sin x = \pm 1/\sqrt{2} = \pm\sqrt{2}/2$

b)  $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$



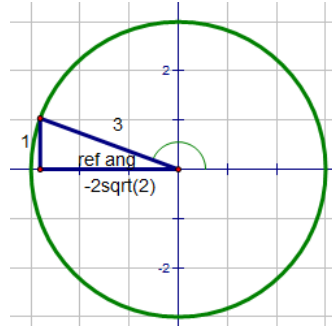
All the ref angles are  $\pi/6$



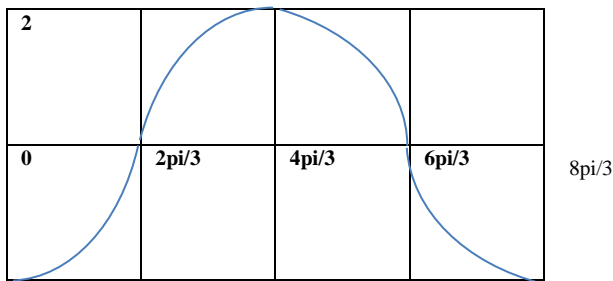
All the ref angles are  $\pi/4$

4. (20 points) Given  $\csc \theta = 3$  and  $\tan \theta < 0$ , find the values of the 5 other trigonometric functions. NO PICTURE NO CREDIT. Use 3 as your radius for circle. Drop or raise the perpendicular to the  $x$ -axis to create a triangle. Draw and label the angle ( $\theta$ ) as well as the reference angle (ref  $\angle$ ).

$$\begin{aligned} \sin \theta &= \frac{1}{3} & \csc \theta &= 3 \\ \cos \theta &= -\frac{2\sqrt{2}}{3} & \sec \theta &= -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4} \\ \tan \theta &= -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4} & \cot \theta &= -2\sqrt{2} \end{aligned}$$

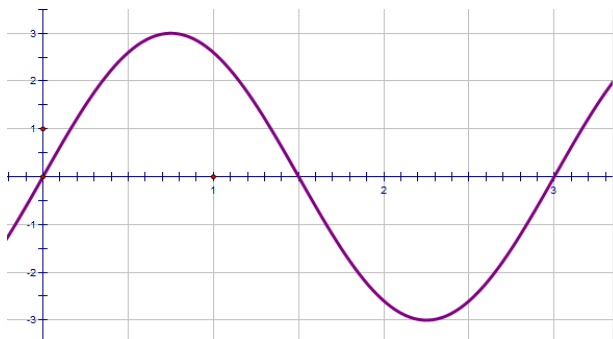


- (14 points) a) Graph 1 period of  $y = -2\cos\left(\frac{3}{4}t\right)$  on axes below. Amplitude is 2 and period is  $2\pi / (3/4) = 8\pi/3$ .



-2

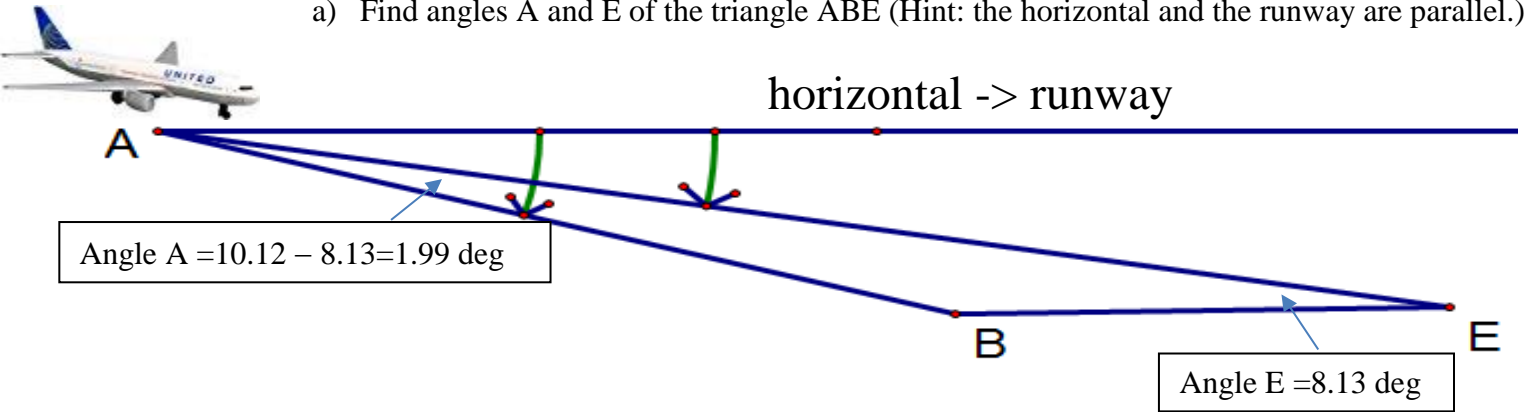
- b) Find the equation for the graph below. Amplitude is 3 and period is 3.



$$\begin{aligned} b &= 2\pi / 3 \\ a &= 3 \\ y &= 3\sin\left(\left(\frac{2\pi}{3}\right)x\right) \end{aligned}$$

5. (20 points) When an airplane A is landing on a 2.03-mile-long runway BE, the angles of depression to the beginning B and end E of the runway are  $10.12^\circ$  and  $8.13^\circ$ , respectively.

a) Find angles A and E of the triangle ABE (Hint: the horizontal and the runway are parallel.)



b) How far is the plane from the near end of the runway (i.e., find the length of AB)?

Round to the nearest **hundredth**. Write your answer as a **sentence** and include **units**.

Using the law of sines:

$$\frac{\sin A}{a} = \frac{\sin E}{e}$$

$$\frac{\sin 1.99^\circ}{2.03} = \frac{\sin 8.13^\circ}{e}$$

$$e = \frac{2.03 \sin 8.13^\circ}{\sin 1.99^\circ} = 8.27 \text{ mi}$$

The plane is 8.27 miles from the beginning of the runway.