Starting with the ratio identity given, use substitution and fundamental identities to write four new identities belonging to the ratio family. Answers may vary.

7. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ 8. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Starting with the Pythagorean identity given, use algebra to write four additional identities belonging to the Pythagorean family. Answers may vary.

9. $1 + \tan^2\theta = \sec^2\theta$ 10. $1 + \cot^2\theta = \csc^2\theta$

Verify the equation is an identity using multiplication and fundamental identities.

11. $\sin\theta\cot\theta = \cos\theta$ 12. $\cos\theta\tan\theta = \sin\theta$

13. $\sec^2\theta\cot^2\theta = \csc^2\theta$ 14. $\csc^2\theta\tan^2\theta = \sec^2\theta$

15. $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

16. $\tan\theta(\cot\theta + \tan\theta) = \sec^2\theta$

17. $\sin\theta(\csc\theta - \sin\theta) = \cos^2\theta$

18. $\cot\theta(\tan\theta + \cot\theta) = \csc^2\theta$

19. $\tan\theta(\csc\theta + \cot\theta) = \sec\theta + 1$

20. $\cot\theta(\sec\theta + \tan\theta) = \csc\theta + 1$

Verify the equation is an identity using factoring and fundamental identities.

21. $\tan^2\theta\csc^2\theta - \tan^2\theta = 1$

22. $\sin^2\theta\cot^2\theta + \sin^2\theta = 1$

23. $\dfrac{\sin\theta\cos\theta + \sin\theta}{\cos\theta + \cos^2\theta} = \tan\theta$

24. $\dfrac{\sin\theta\cos\theta + \cos\theta}{\sin\theta + \sin^2\theta} = \cot\theta$

25. $\dfrac{1 + \sin\theta}{\cos\theta + \cos\theta\sin\theta} = \sec\theta$

26. $\dfrac{1 + \cos\theta}{\sin\theta + \cos\theta\sin\theta} = \csc\theta$

27. $\dfrac{\sin\theta\tan\theta + \sin\theta}{\tan\theta + \tan^2\theta} = \cos\theta$

28. $\dfrac{\cos\theta\cot\theta + \cos\theta}{\cot\theta + \cot^2\theta} = \sin\theta$

Verify the equation is an identity using special products and fundamental identities.

29. $\dfrac{(\sin\theta + \cos\theta)^2}{\cos\theta} = \sec\theta + 2\sin\theta$

30. $\dfrac{(1 + \tan\theta)^2}{\sec\theta} = \sec\theta + 2\sin\theta$

31. $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$

32. $(\sec\theta + 1)(\sec\theta - 1) = \tan^2\theta$

33. $\dfrac{(\csc\theta - \cot\theta)(\csc\theta + \cot\theta)}{\tan\theta} = \cot\theta$

34. $\dfrac{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\csc\theta} = \sin\theta$

Verify the equation is an identity using fundamental identities and $\dfrac{A}{B} \pm \dfrac{C}{D} = \dfrac{AD \pm BC}{BD}$ to combine terms.

35. $\dfrac{\cos^2\theta}{\sin\theta} + \dfrac{\sin\theta}{1} = \csc\theta$

36. $\dfrac{\sec\alpha}{1} - \dfrac{\tan^2\alpha}{\sec\alpha} = \cos\alpha$

37. $\dfrac{\tan\theta}{\csc\theta} - \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta - 1}{\cot\theta}$

38. $\dfrac{\cot\theta}{\sec\theta} - \dfrac{\cos\theta}{\sin\theta} = \dfrac{\cos\theta - 1}{\tan\theta}$

39. $\dfrac{\sec\theta}{\sin\theta} - \dfrac{\csc\theta}{\sec\theta} = \tan\theta$ 40. $\dfrac{\csc\theta}{\cos\theta} - \dfrac{\sec\theta}{\csc\theta} = \cot\theta$

Write the given function entirely in terms of the second function indicated.

41. $\tan\theta$ in terms of $\sin\theta$ 42. $\tan\theta$ in terms of $\sec\theta$

43. $\sec\theta$ in terms of $\cot\theta$ 44. $\sec\theta$ in terms of $\sin\theta$

45. $\cot\theta$ in terms of $\sin\theta$ 46. $\cot\theta$ in terms of $\csc\theta$

For the function $f(\theta)$ and the quadrant in which θ terminates, state the value of the other five trig functions.

47. $\cos\theta = -\dfrac{20}{29}$ with θ in QII

48. $\sin\theta = \dfrac{12}{37}$ with θ in QII

49. $\tan\theta = \dfrac{15}{8}$ with θ in QIII

50. $\sec\theta = \dfrac{45}{27}$ with θ in QIV

51. $\cot\theta = \dfrac{x}{5}$ with θ in QI

52. $\csc\theta = \dfrac{7}{x}$ with θ in QII

53. $\sin\theta = -\dfrac{7}{13}$ with θ in QIII

54. $\cos\theta = \dfrac{23}{25}$ with θ in QIV

55. $\sec\theta = -\dfrac{9}{7}$ with θ in QII

56. $\cot\theta = -\dfrac{11}{2}$ with θ in QIV

11. $\dfrac{1 - \sin(-x)}{\cos x + \cos(-x)\sin x} = \sec x$

12. $\dfrac{1 + \cos(-x)}{\sin x - \cos x \sin(-x)} = \csc x$

13. $\cos^2 x \tan^2 x = 1 - \cos^2 x$

14. $\sin^2 x \cot^2 x = 1 - \sin^2 x$

15. $\tan x + \cot x = \sec x \csc x$

16. $\cot x \cos x = \csc x - \sin x$

17. $\dfrac{\cos x}{\tan x} = \csc x - \sin x$

18. $\dfrac{\sin x}{\cot x} = \sec x - \cos x$

19. $\dfrac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$

20. $\dfrac{\sin \theta}{1 - \cos \theta} = \csc \theta + \cot \theta$

21. $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

22. $\dfrac{1 - \cos x}{\sin x} = \dfrac{\sin x}{1 + \cos x}$

23. $\dfrac{\csc x}{\cos x} - \dfrac{\cos x}{\csc x} = \dfrac{\cot^2 x + \sin^2 x}{\cot x}$

24. $\dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x} = \csc^2 x \sec^2 x$

25. $\dfrac{\sin x}{1 + \sin x} - \dfrac{\sin x}{1 - \sin x} = -2 \tan^2 x$

26. $\dfrac{\cos x}{1 + \cos x} - \dfrac{\cos x}{1 - \cos x} = -2 \cot^2 x$

27. $\dfrac{\cot x}{1 + \csc x} - \dfrac{\cot x}{1 - \csc x} = 2 \sec x$

28. $\dfrac{\tan x}{1 + \sec x} - \dfrac{\tan x}{1 - \sec x} = 2 \csc x$

29. $\dfrac{\sec^2 x}{1 + \cot^2 x} = \tan^2 x$

30. $\dfrac{\csc^2 x}{1 + \tan^2 x} = \cot^2 x$

31. $\sin^2 x\,(\cot^2 x - \csc^2 x) = -\sin^2 x$

32. $\cos^2 x\,(\tan^2 x - \sec^2 x) = -\cos^2 x$

33. $\cos x \cot x + \sin x = \csc x$

34. $\sin x \tan x + \cos x = \sec x$

35. $\dfrac{\sec x}{\cot x + \tan x} = \sin x$

36. $\dfrac{\csc x}{\cot x + \tan x} = \cos x$

37. $\dfrac{\sin x - \csc x}{\csc x} = -\cos^2 x$

38. $\dfrac{\cos x - \sec x}{\sec x} = -\sin^2 x$

39. $\dfrac{1}{\csc x - \sin x} = \tan x \sec x$

40. $\dfrac{1}{\sec x - \cos x} = \cot x \csc x$

41. $\dfrac{1 + \sin x}{1 - \sin x} = (\tan x + \sec x)^2$

42. $\dfrac{1 - \cos x}{1 + \cos x} = (\csc x - \cot x)^2$

43. $\dfrac{\cos x - \sin x}{1 - \tan x} = \dfrac{\cos x + \sin x}{1 + \tan x}$

44. $\dfrac{1 - \cot x}{1 + \cot x} = \dfrac{\sin x - \cos x}{\sin x + \cos x}$

45. $\dfrac{\tan^2 x - \cot^2 x}{\tan x - \cot x} = \csc x \sec x$

46. $\dfrac{\cot x - \tan x}{\cot^2 x - \tan^2 x} = \sin x \cos x$

47. $\dfrac{\cot x}{\cot x + \tan x} = 1 - \sin^2 x$

48. $\dfrac{\tan x}{\cot x + \tan x} = 1 - \cos^2 x$

49. $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = 1$

50. $\dfrac{\csc^4 x - \cot^4 x}{\csc^2 x + \cot^2 x} = 1$

51. $\dfrac{\cos^4 x - \sin^4 x}{\cos^2 x} = 2 - \sec^2 x$

52. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x} = 2 - \csc^2 x$

53. $(\sec x + \tan x)^2 = \dfrac{(\sin x + 1)^2}{\cos^2 x}$

54. $(\csc x + \cot x)^2 = \dfrac{(\cos x + 1)^2}{\sin^2 x}$