EXERCISE SET 3-2

Writing Exercises

1. Your boyfriend/girlfriend randomly opens your math book and ends up seeing a truth table in Section 3-2. How would you explain to him or her what the point of a truth table is?
2. Explain the difference between the inclusive and exclusive disjunctions. Write an example of each in plain English, and explain why each is the type of disjunction described.
3. I claim that a biconditional statement is really a conjunction of two conditional statements. Explain why that makes sense.
4. Describe the hierarchy of connectives. What's the point of having one? How do you use it?

Computational Exercises

For Exercises 5–34, construct a truth table for each.

5. $\sim(p \vee q)$
6. $q \rightarrow p$
7. $\sim p \wedge q$
8. $\sim q \rightarrow \sim p$
9. $\sim p \leftrightarrow q$
10. $(p \vee q) \rightarrow \sim p$
11. $\sim(p \wedge q) \rightarrow p$
12. $(p \vee q) \wedge (q \vee p)$
13. $(\sim q \wedge p) \rightarrow \sim p$
14. $q \wedge \sim p$
15. $(p \wedge q) \leftrightarrow (q \vee \sim p)$
16. $p \rightarrow (q \vee \sim p)$
17. $(p \wedge q) \vee p$
18. $(q \rightarrow p) \vee \sim r$
19. $(r \wedge q) \vee (p \wedge q)$
20. $(r \rightarrow q) \vee (p \rightarrow r)$
21. $\sim(p \vee q) \rightarrow \sim(p \wedge r)$
22. $(\sim p \vee \sim q) \rightarrow \sim r$
23. $(\sim p \vee q) \wedge r$
24. $p \wedge (q \vee \sim r)$
25. $(p \wedge q) \leftrightarrow (\sim r \vee q)$
26. $\sim(p \wedge r) \rightarrow (q \wedge r)$
27. $r \rightarrow \sim(p \vee q)$
28. $(p \vee q) \vee (\sim p \vee \sim r)$
29. $p \rightarrow (\sim q \wedge \sim r)$
30. $(q \vee \sim r) \leftrightarrow (p \wedge \sim q)$
31. $\sim(q \rightarrow p) \wedge r$
32. $q \rightarrow (p \wedge r)$
33. $(r \vee q) \wedge (r \wedge p)$
34. $(p \wedge q) \leftrightarrow \sim r$

If p and r are false statements, and q is a true statement, find the truth value of each compound statement in Exercises 35–40.

35. $q \vee (p \wedge \sim r)$
36. $(p \wedge q) \vee (q \wedge r)$
37. $r \rightarrow \sim(p \vee q)$
38. $\sim(p \wedge q) \vee \sim r$
39. $\sim p \wedge \sim(r \vee \sim q)$
40. $(p \rightarrow r) \rightarrow (\sim q \wedge p)$

Applications in Our World

For Exercises 41–46, use the truth value of each simple statement to determine the truth value of the compound statement. Use the Internet if you need help determining the truth value of a simple statement.

41. p: Japan bombs Pearl Harbor.
    q: the United States stays out of World War II.
    Statement: $p \rightarrow q$
42. p: Barack Obama wins the Democratic nomination in 2008.
    q: Mitt Romney wins the Republican nomination in 2008.
    Statement: $p \wedge q$
43. p: NASA sends a manned spacecraft to the Moon.
    q: NASA sends a manned spacecraft to Mars.
    Statement: $p \vee q$
44. p: an oppressive regime is overthrown in Egypt in 2011.
    q: free and open elections are held in Egypt in 2011.
    Statement: $p \rightarrow q$
45. p: Apple releases the iPad.
    q: Apple stops making desktop computers.
    r: Samsung and Amazon release tablet computers.
    Statement: $(p \vee q) \wedge r$
46. p: a large earthquake and tsunami devastate Japan.
    q: some nuclear reactors in Japan are damaged.
    r: Japan stops using nuclear power plants.
    Statement: $p \wedge q \rightarrow \sim r$

Exercises 47–52 are based on the compound statement below.

A new weight loss supplement claims that if you take the product daily and cut your calorie intake by 10%, you will lose at least 10 pounds in the next 4 months.

47. This compound statement is made up of three simple statements. Identify them and assign a letter to each.
48. Write the compound statement in symbolic form, using conjunctions and the conditional.
49. Construct a truth table for the compound statement you wrote in Exercise 48.
50. If you take this product daily and don't cut your calorie intake by 10%, and then don't lose 10 pounds, is the claim made by the advertiser true or false?
51. If you take the product daily, don't cut your calorie intake by 10%, and do lose 10 pounds, is the claim true or false?
52. If you don't take the product daily, cut your calorie intake by 10%, and do lose 10 pounds, is the claim true or false?

Exercises 53–58 are based on the compound statement below.

The owner of a professional baseball team publishes an open letter to fans after another losing season. He claims

that if attendance for the following season is over 2 million, then he will add $20 million to the payroll and the team will make the playoffs the following year.

53. This compound statement is made up of three simple statements. Identify them and assign a letter to each.
54. Write the compound statement in symbolic form, using conjunction and the conditional.
55. Construct a truth table for the compound statement you wrote in Exercise 54.
56. If attendance goes over 2 million the next year and the owner raises payroll by $20 million, but the team fails to make the playoffs, is the owner's claim true or false?
57. If attendance is less than 2 million but the owner still raises the payroll by $20 million and the team makes the playoffs, is the owner's claim true or false?
58. If attendance is over 2 million, the owner doesn't raise the payroll, but the team still makes the playoffs, is the owner's claim true or false?

Critical Thinking

59. Construct two truth tables to show that the statement $p \wedge q \vee r$ is ambiguous. (*Hint*: Look back at our discussion of the hierarchy of connectives.)
60. Let's look a little deeper at the statement $p \wedge q \vee r$. Write three simple statements for p, q, and r so that $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ have different meanings. (*Hint*: The truth tables from Exercise 59 will probably help.)
61. Using the hierarchy for connectives, write the statement $p \rightarrow q \vee r$ by using parentheses to indicate the proper order. Then construct truth tables for $(p \rightarrow q) \vee r$ and $p \rightarrow (q \vee r)$. Are the resulting truth values the same? Are you surprised? Why or why not?
62. In 2003, New York City Council was considering banning indoor smoking in bars and restaurants. Opponents of the ban claimed that it would have a negligible effect on indoor pollution, but a huge negative effect on the economic success of these businesses. Eventually, the ban was enacted, and a 2004 study by the city department of health found that there was a sixfold decrease in indoor air pollution in bars and restaurants, but jobs, liquor licenses, and tax revenues all increased. Assign truth values to all the premises of the opponents' claim; then write the claim as a compound statement and determine its validity.
63. Consider the following two statements:

    "If we scored more points, we won!"
    "We didn't score more points, or we won."

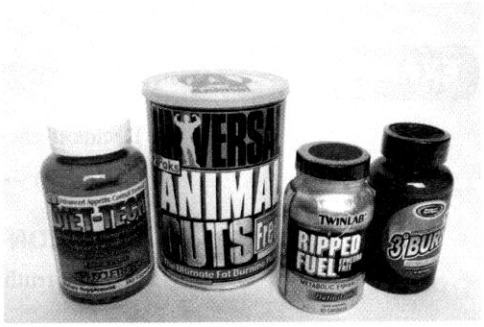
    Explain why these statements say exactly the same thing from an English standpoint. (We'll deal with them logically in the next question.)
64. Write the two statements in Exercise 63 as compound statements using letters p and q, then construct a truth table for each compound statement. How can you use this to help convince you that the truth table we built for the conditional is correct?

Section 3-3 Types of Statements

LEARNING OBJECTIVES

- ❑ 1. Classify a statement as a tautology, a self-contradiction, or neither.
- ❑ 2. Identify logically equivalent statements.
- ❑ 3. Write negations of compound statements.
- ❑ 4. Write the converse, inverse, and contrapositive of a statement.

It's no secret that weight loss has become big business in the United States. It seems like almost every week, a new company pops into existence with the latest miracle pill to turn you into a supermodel.



A typical advertisement will say something like "Use of our product may result in significant weight loss." That sounds great, but think about what that statement really means. If use of the product "may" result in significant weight loss, it also may not result in any weight loss at all, it may result in weight gain, or it may result in turning you into a pumpkin. In fact, the statement could be translated into "You will lose weight or you will not lose weight." Of course, this statement is always true. In this section, we'll study statements of this type (and others).