Remember that both inductive reasoning and deductive reasoning are useful tools for problem solving. But the biggest difference between them is that conclusions drawn from inductive reasoning, no matter how reasonable, are still at least somewhat uncertain. In Problems 69–74, we'll distinguish between *weak* and *strong* inductive arguments. But conclusions drawn by using deductive reasoning can be considered definitely true, as long as the general rules used to draw the conclusion are known to be true.

In addition, you should take a minute or two to think about the fact that to disprove a conjecture, you only need to find *one specific example* for which it's not true. But to prove a conjecture, you have to show that it's true in *every* possible case.

Answers to Try This One

1 Pattern: every entry is 1 more than the one that comes two spots before it. The next three numbers are 8, 6, 9.

2 

3 Always odd

4 True

5 June has neither a y nor an r.

6 There are 57.

7 10

8 The result is the original number.

9 Deductive

10 Inductive

EXERCISE SET 1-1

Writing Exercises

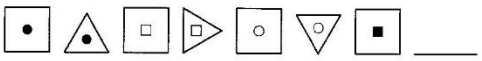
1. Explain the difference between inductive and deductive reasoning.
2. What is meant by the term *conjecture?*
3. Give an example of a decision you made based on inductive reasoning that turned out well, and one that turned our poorly.
4. What is a counterexample? What are counterexamples used for?
5. Explain why you can never be sure that a conclusion you arrived at using inductive reasoning is true.
6. Explain the difference between an arbitrary number and a number selected at random.

Computational Exercises

For Exercises 7–16, use inductive reasoning to find a pattern, and then make a reasonable conjecture for the next number or item in the sequence.

7. 1 2 4 7 11 16 22 29 __
8. 6 10 22 58 166 490 __
9. 10 20 11 18 12 16 13 14 14 12 15 __
10. 2 3 8 63 3,968 __
11. 100 99 97 94 90 85 79 __
12. 9 12 11 14 13 16 15 18 __

13.  _____

14.  _____

15.  _____

16.  _____

For Exercises 17–20, find a counterexample to show that each statement is false.

17. The sum of any three odd numbers is even.
18. When an even number is added to the product of two odd numbers, the result will be even.
19. When an odd number is squared and divided by 2, the result will be a whole number.

20. When any number is multiplied by 6 and the digits of the answer are added, the sum will be divisible by 6.

For Exercises 21–24, use inductive reasoning to make a conjecture about a rule that relates the number you selected to the final answer. Try to prove your conjecture by using deductive reasoning.

21. Pick a number:
    Double it:
    Subtract 20 from the answer:
    Divide by 2:
    Subtract the original number:
    Result:
22. Pick a number:
    Multiply it by 9:
    Add 21:
    Divide by 3:
    Subtract three times the original number:
    Result:
23. Pick a number:
    Add 6:
    Multiply the answer by 9:
    Divide the answer by 3:
    Subtract 3 times the original number:
    Result:
24. Pick an even number:
    Multiply it by 4:
    Add 8 to the product:
    Divide the answer by 2:
    Subtract 2 times the original number:
    Result:

For Exercises 25–34, use inductive reasoning to find a pattern for the answers. Then use the pattern to guess the result of the final calculation, and perform the operation to see if your answer is correct.

25. $12{,}345{,}679 \times 9 = 111{,}111{,}111$
    $12{,}345{,}679 \times 18 = 222{,}222{,}222$
    $12{,}345{,}679 \times 27 = 333{,}333{,}333$
    $\vdots$
    $12{,}345{,}679 \times 72 = ?$
26. $0^2 + 1 = 1$
    $1^2 + 3 = 2^2$
    $2^2 + 5 = 3^2$
    $3^2 + 7 = 4^2$
    $4^2 + 9 = 5^2$
    $5^2 + 11 = ?$
27. $999{,}999 \times 1 = 0{,}999{,}999$
    $999{,}999 \times 2 = 1{,}999{,}998$
    $999{,}999 \times 3 = 2{,}999{,}997$
    $\vdots$
    $999{,}999 \times 9 = ?$
28. $1 = 1^2$
    $1 + 2 + 1 = 2^2$
    $1 + 2 + 3 + 2 + 1 = 3^2$
    $\vdots$
    $1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = ?$
29. $9 \times 9 = 81$
    $99 \times 99 = 9{,}801$
    $999 \times 999 = 998{,}001$
    $9{,}999 \times 9{,}999 = 99{,}980{,}001$
    $99{,}999 \times 99{,}999 = ?$
30. $1 \times 8 + 1 = 9$
    $12 \times 8 + 2 = 98$
    $123 \times 8 + 3 = 987$
    $1{,}234 \times 8 + 4 = 9{,}876$
    $12{,}345 \times 8 + 5 = ?$
31. $1 \cdot 1 = 1$
    $11 \cdot 11 = 121$
    $111 \cdot 111 = 12{,}321$
    $1{,}111 \cdot 1{,}111 = 1{,}234{,}321$
    $11{,}111 \cdot 11{,}111 = ?$
32. $9 \cdot 91 = 819$
    $8 \cdot 91 = 728$
    $7 \cdot 91 = 637$
    $6 \cdot 91 = 546$
    $5 \cdot 91 = ?$
33. Explain what happens when the number 142,857 is multiplied by the numbers 2 through 8.
34. A Greek mathematician named Pythagoras is said to have discovered the following number pattern. Find the next three sums by using inductive reasoning. Don't just add!
    $1 = 1$
    $1 + 3 = 4$
    $1 + 3 + 5 = 9$
    $1 + 3 + 5 + 7 = 16$
    $1 + 3 + 5 + 7 + 9 = ?$
    $1 + 3 + 5 + 7 + 9 + 11 = ?$
    $1 + 3 + 5 + 7 + 9 + 11 + 13 = ?$
35. Use inductive reasoning to make a conjecture about the next three sums, and then perform the calculations to verify that your conjecture is true.
    $1 + \frac{1}{2} = \frac{3}{2}$
    $1 + \frac{1}{2} + \frac{1}{2 \cdot 3} = \frac{5}{3}$
    $1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{7}{4}$
    $1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} = ?$
    $1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} = ?$
    $1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} = ?$
36. Use inductive reasoning to determine the unknown sum, then perform the calculation to verify your answer.
    $2 = 1(2)$
    $2 + 4 = 2(3)$
    $2 + 4 + 6 = 3(4)$
    $2 + 4 + 6 + 8 = 4(5)$
    $2 + 4 + 6 + 8 + 10 + 12 + 14 = ?$

In Exercises 37–40, use inductive reasoning to find a pattern, then make a reasonable conjecture for the next three items in the pattern.

37. d b e c f d __ __ __
38. a b c e d f i g h o __ __ __
39. J F M A __ __ __
40. D N O S A __ __ __

Applications in Our World

In Exercises 41–58, determine whether the type of reasoning used is inductive or deductive reasoning.

41. The last four congressional representatives from this district were all Republicans. I don't know why the Democratic candidate is even bothering to run this year.
42. I know I will have to work a double shift today because I have a migraine and every time I have a migraine I get stuck pulling a double.
43. If class is canceled, I go to the beach with my friends. I didn't go to the beach with my friends yesterday; so class was not canceled.
44. On Christmas Day, movie theaters and Chinese restaurants are always open, so this Christmas Day we can go to a movie and get some Chinese takeout.
45. For the first three games this year, the parking lot was packed with tailgaters, so we'll have to leave extra early to find a spot this week.
46. Every time Beth sold back her textbooks, she got about 10% of what she paid for them; so this semester she realized it would not be worth the effort to sell back her books at all.
47. Experts say that opening email attachments that come from unknown senders is the easiest way to get a virus on your computer. Shauna constantly opens attachments from people she doesn't know, so she'll probably end up with a virus on her system.
48. Whenever Marcie let friends set her up on a blind date, the guy turned out to be a total loser. This time, when a friend offered to fix her up, she decided the guy would be a loser, so she declined.
49. Dr. Spalsbury's policy is that any student whose cell phone goes off during class will be asked to leave. So when Ericka forgot to turn hers off and it rang during a quiz, out the door she went.
50. While experimenting on learning in mice, a biology student was able to successfully train six different mice to finish a maze, so she was really surprised when the next one was unable to learn the maze.
51. Since Josie ate a diet of mostly foods high in saturated fat, she was not surprised when her doctor said her cholesterol levels were too high.
52. In the past, even when Chris followed a recipe, her meal was either burned or underdone. Now her party guests know to eat before they attend her dinners so they won't starve all evening.
53. Working as a nurse in a hospital requires at least a two-year degree in this state, so when I was in the emergency room last week I asked the nurse where he went to college.
54. Marathon runners should eat extra carbs before a big race, and since Mark did not eat enough carbs before the race, he felt sluggish the entire time.
55. Organizing chapter contents in your own words before the test will decrease the amount of study you have to do before a test. When Scott tried this method, he was pleasantly surprised at how fast he was able to study.
56. The last several network dramas I've followed have been canceled just when I started getting into them. So I'm not going to bother watching the new one they're advertising even though it looks good, because I don't want to be disappointed when it gets canceled.
57. Rollerblading without knee pads and a helmet is said to be dangerous. So when I got my first pair of Rollerblades, I made sure to get a helmet and knee pads.
58. Whenever Sarah drove over the speed bumps near the dorm too fast, her CD player would skip around like crazy. So this time before she entered the dorm parking lot, she paused the CD.

Critical Thinking

59. Do a Google search for the string "studies texting while driving." Suppose that you've driven while texting ten times in the past without any incident. How likely would you be to text while driving if you use (a) inductive reasoning, and (b) deductive reasoning based on your Google search? Describe your reasoning in each case.
60. Just about everyone had a conversation like this with their parents at some point in their childhood: "But all my friends are doing it!" "If your friends jumped off a bridge, would you jump too?" Describe how arguments like this apply to inductive and deductive reasoning. Specifically, what type of reasoning is each person using, and who in your opinion makes a stronger argument?
61. (a) Find a likely candidate for the next two numbers in the following sequence: 2, 4, 8, . . .