Solving trigonometric equations:

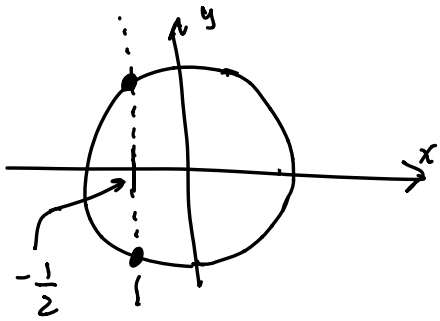
Example: Find all solutions of

$$\cos(u) = -\frac{1}{2}, \; u \text{ in } [0, 2\pi).$$

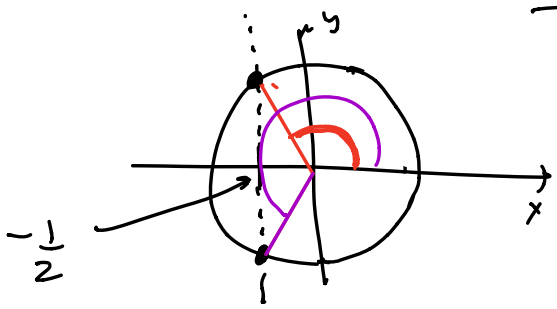
To find Intersecion points

$$\boxed{\text{Solve } \begin{cases} x = -\frac{1}{2} \\ x^2 + y^2 = 1 \end{cases}}$$

Note 2 solutions which give no terminal sides to our solutions

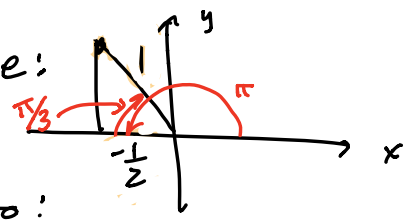


This means the x-coordinate of our point on the unit circle is $-\frac{1}{2}$.

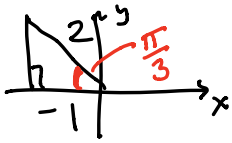


There are 2 solutions. One indicated in red and the other in purple.

Consider the red one:



This is similar to:



which is recognized as a special angle with reference angle 60° or $\frac{\pi}{3}$.

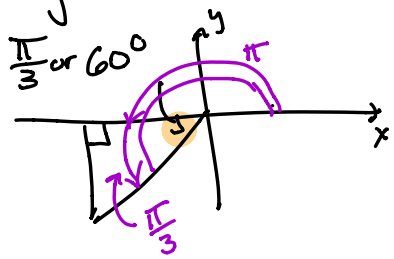
So, the red angle is

$$180^\circ - 60^\circ = 120^\circ$$

or

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

For the purple one, the reference angle is the same as the red:



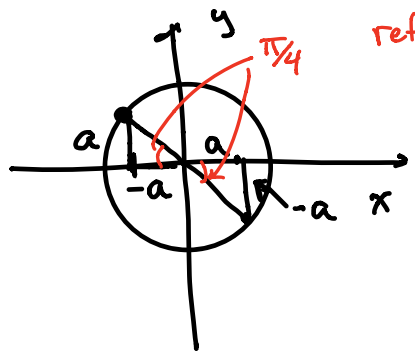
So the angle is

$180^\circ + 60^\circ = 240^\circ$

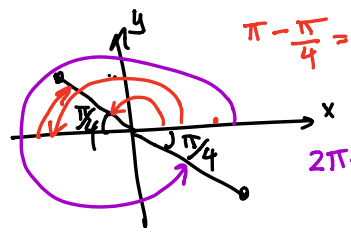
or $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

So, there are 2 solutions in $[0, 2\pi)$: $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

<u>Example</u>: $\tan u = -1$, $u \in [0, 2\pi)$



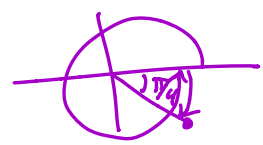
reference angle is $\frac{\pi}{4}$.



$\pi - \frac{\pi}{4} = \frac{4\pi}{4} - \frac{1\pi}{4} = \frac{3\pi}{4}$

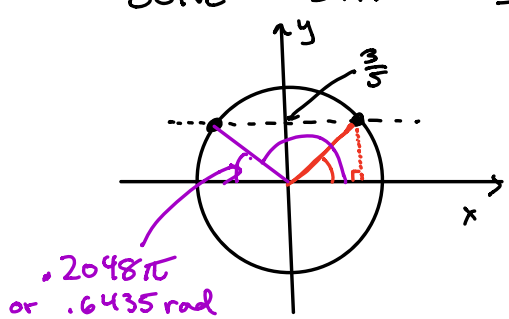
$2\pi - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4}$

$= \frac{7\pi}{4}$



The solutions are $\frac{3\pi}{4}$ & $\frac{7\pi}{4}$.

Solve $\sin u = \frac{3}{5}$ for u in $[0, 2\pi)$.



Use the calculator to find the reference angle:

$\text{Arcsin}\left(\frac{3}{5}\right) = 36.87^\circ$

$= .6435$ rad

$\approx \pi - .2048\pi \approx .7952\pi$

or 2.498

$\left(= \frac{.6435}{\pi}\pi \text{ rad} = 0.2048\,\pi. \right)$

So, we have 2 solutions:

$.6435$ and 2.498

or, 0.2048π and 0.7952π

Remark: Arcsin a is in $(-\pi/2, \pi/2]$

Arccos a is in $[0, \pi)$.

& Arctan a is in $(-\frac{\pi}{2}, \pi/2)$.

Other solutions must be obtained by hand.