GCF and factoring

Recall the process of distribution:

Ex: $3x^2y^4(1-2xy^2+3x^2y^3)$

$= 3x^2y^4 - 6x^3y^6 + 9x^4y^7$

Now we want to "go backwards"

Example: Starting from a much more modest question, what is the greatest common factor (gcf) $2\cdot 3^2\cdot 5^2$ and $2^2\cdot 3^3\cdot 5$?

In other words, what is the largest number that divides the two numbers?

It is clear that 2 divides 2 and 2^2
but 2^2 doesn't divide 2.
So, 2 is the gcf of 2 and 2^2.

It is clear that 3^2 divides 3^2 and 3^3
but 3^3 does not divide 3^2.
So, 3^2 is the gcf of 3^2 & 3^3.

Similarly, 5 is the gcf of 5 and 5^2. (why?)

So gcf of $2\cdot 3^2\cdot 5^2$ and $2^2\cdot 3^3\cdot 5$
is $2\cdot 3^2\cdot 5$ $(= 90)$.

Example: What is the gcf of x^2, x^3, and x^4?

We see x^2 is a factor of x^2, x^3, & x^4.
but x^3 is not a factor of x^2.
since $\frac{x^2}{x^3} = \frac{1}{x}$ ∴ So, x^2 is the gcf.

Example: GCF of $3x^2y^4$, $6x^3y^6$ & $9x^4y^7$ is $3x^2y^4$.

Example GCF of $10x^2y^3$ and $15x^3y$ is $5x^2y$.

Factoring out the GCF

Example:

$$3x^2y^4 - 6x^3y^6 + 9x^4y^7 = 3x^2y^4(1 - 2xy^2 + 3x^2y^3)$$

since $$\frac{3x^2y^4 - 6x^3y^6 + 9x^4y^7}{3x^2y^4}$$

$$= \frac{3x^2y^4}{3x^2y^4} - \frac{6x^3y^6}{3x^2y^4} + \frac{9x^4y^7}{3x^2y^4}$$

$$= 1 - 2xy^2 + 3x^2y^3.$$

① Try: Factor out the GCF:

$(10x^2y^3 - 15x^3y)$

② Try: Factor out the GCF

$5x^6y^9 - 10x^7y^6 + 5x^3y^5$

Factoring by grouping

Example: $x^2 + 3x + 2x + 6$

① First group in pairs of terms

$$(x^2 + 3x) + (2x + 6)$$

② Factor out GCF for each binomial:

$$x(x + 3) + 2(x + 3)$$

③ Note the common factor and factor it out.

$$(x+3)(x+2)$$

Example: Factor by grouping:

$$12x^2 + 10x - 18x - 15$$

① $(12x^2 + 10x) + (-18x - 15)$

② $2x(6x+5) + (-3)(6x+5)$

③ $(6x+5)(2x-3)$

chose -3 so we'd have 6x+5 in common.

③ **Try:** Factor by grouping

$$27x^2 + 18x - 6x - 4$$

Ans

① $5x^2y(2y^2-3x)$

② $5x^3y^5(x^2y^4-2x^4y+1)$

③ $(3x+2)(9x-2)$