Complex Fractions: Examples of 2 Methods for Simplification.

Example

Consider $\dfrac{\left(\dfrac{3z}{2xy}\right)}{\left(\dfrac{3x}{10yz}\right)}$.

We want to simplify:

Method 1

Divide:

$$\frac{\left(\frac{3z}{2xy}\right)}{\left(\frac{3x}{10yz}\right)} = \frac{3z}{2xy}\cdot\frac{10yz}{3x}$$

multiply

$$= \frac{3z\cdot 10yz}{2xy\cdot 3x}$$

reduce

$$= \frac{z\cdot 5\cdot 2\cdot z}{2\cdot x\cdot x}$$

$$= \frac{5z^2}{x^2}$$

Both are valid when $y\neq 0$.

Method 2

Multiply Numerator and denominator by the LCM of $2xy$ & $10yz$ which is $10xyz$:

$$\frac{\left(\frac{3z}{2xy}\right)}{\left(\frac{3x}{10yz}\right)}\cdot\frac{\frac{5\cdot 2xyz}{1}}{\frac{5\cdot 2xyz}{1}}$$

$$= \frac{\left(\frac{3z\,5\cdot 2xyz}{2xy}\right)}{\left(\frac{3x\cdot 5\cdot 2x\cdot y\cdot z}{10yz}\right)}$$

reduce

$$= \frac{3z\cdot 5z}{3x\cdot x}$$

reduce

$$= \frac{5z^2}{x^2}$$

Example:

$$\frac{\frac{1}{z^2} + \frac{3}{z}}{\frac{2}{z} - \frac{5}{z^3}}$$

Method 1:

$$\frac{\frac{1}{z^2} + \frac{3}{z}}{\frac{2}{z} - \frac{5}{z^3}} =$$

Add fractions in Num. & Denom.

$$\frac{\frac{1}{z^2} + \frac{3z}{zz}}{\frac{z^2 2}{z^2 z} - \frac{5}{z^3}} = \frac{\left(\frac{1+3z}{z^2}\right)}{\left(\frac{2z^2-5}{z^3}\right)}$$

Divide

$$= \frac{1+3z}{z^2} \cdot \frac{z^3}{2z^2-5}$$

$$= \frac{(1+3z)(z^3)}{z^2(2z^2-5)}$$

reduce

$$= \frac{z(1+3z)}{2z^2-5}$$

Valid for $z \neq 0$.

Method 2

LCM of z^2, z, z^3 is z^3:

$$\frac{\frac{1}{z^2} + \frac{3}{z}}{\frac{2}{z} - \frac{5}{z^3}} \cdot \frac{\frac{z^3}{1}}{\frac{z^3}{1}}$$

$$= \frac{\frac{z^3}{z^2} + \frac{3z^3}{z}}{\frac{2z^3}{z} - \frac{5z^3}{z^3}}$$

Simplify

$$= \frac{z + 3z^2}{2z^2-5}$$

$$\left(= \frac{z(1+3z)}{2z^2-5}\right)$$