Radical Equations.

Example: $\sqrt{x} = 4$. It is easy to see that x must be 16, but we could also see that

$\sqrt{x} = 4 \Rightarrow x = 16$. (since $(\sqrt{x})^2 = x$)

$(\sqrt{x})^2 = 4^2$

Example: $\sqrt{x-1} = 2$. It is not hard to see $x = 5$ since $\sqrt{5-1} = \sqrt{4} = 2$.

We can also see it this way:

$$\sqrt{x-1} = 2 \Rightarrow (\sqrt{x-1})^2 = 2^2$$
$$\Rightarrow x - 1 = 4$$
$$\Rightarrow x = 5. \quad \sqrt{5-1} = 2 \text{ is true.}$$

Example: $\sqrt{x-1} + 3 = 15$. It is hard to see what the solution is without work!

isolate $\sqrt{*}$ $\longrightarrow$ $\sqrt{x-1} + 3 = 15 \Rightarrow \sqrt{x-1} = 12$

$$\Rightarrow x - 1 = 12^2 \Rightarrow x = 145.$$

Example: $x - \sqrt{3x-5} = 1$. Almost impossible to see without work!

Isolate $\sqrt{*}$ $\longrightarrow$ $-\sqrt{3x-5} = 1 - x$

$$\Rightarrow (-\sqrt{3x-5})^2 = (1-x)^2 = (1-x)(1-x)$$

This is a quadratic equation! $\longrightarrow$ $\Rightarrow 3x - 5 = 1 - 2x + x^2$

$$\Rightarrow x^2 - 5x + 6 = 0$$
$$\Rightarrow (x-3)(x-2) = 0$$
$$\Rightarrow x = 2 \text{ or } 3$$

This says: **If** $x - \sqrt{3x-5} = 1$ then $x = 2$ or 3.

It does not say $x = 2$ & $x = 3$ are solutions!

We *must* check: $2 - \sqrt{2 \cdot 3 - 5} = 2 - \sqrt{1} = 1$ ✓

& $3 - \sqrt{3 \cdot 3 - 5} = 3 - \sqrt{4} = 1$ ✓

So 2 and 3 are solutions.

Example: $\sqrt{x} = -2 \Rightarrow x = (-2)^2 = 4$

But $\sqrt{4} = 2 \neq -2$. So, there are no solutions.

Example: $2 = x - \sqrt{6-5x}$

$\Rightarrow 2 - x = -\sqrt{6-5x}$

$\Rightarrow (2-x)^2 = 6 - 5x$

$\Rightarrow x^2 - 4x + 4 = 6 - 5x$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x + 2 = 0$ or

$x - 1 = 0$

$\Rightarrow x = -2$ or 1.

But $-2 - \sqrt{6 - 5\cdot(-2)} = -6 \neq 2$

and $1 - \sqrt{6-5} = 0 \neq 2$.

So there are <u>no</u> solutions.

Example: $x-2=2\sqrt{x+1}-x$

$$\Rightarrow 2x-2=2\sqrt{x+1}$$

$$\Rightarrow x-1=\sqrt{x+1}$$

$$\Rightarrow x^2-2x+1=x+1$$

$$\Rightarrow x^2-3x=0$$

$$\Rightarrow x(x-3)=0$$

$$\Rightarrow x=0 \text{ or } 3.$$

But while $3-2=2\sqrt{3+1}-3$,

$$-2\neq 2\sqrt{0+1}-0.$$

So, there is one solution:

$$x=3.$$