Series-Parallel DC Circuits

Lab 001

Electrical Networks

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CET 3525 Section E213
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Table of Contents

Objective

The objective of this laboratory exercise is to understand the characteristics of series-parallel networks and verify the theoretical results by comparing them to measured values obtained upon construction of said networks, verifying the validity of *Ohm's Law* by observation of the relationship between voltage, current, and resistance. Students are expected to follow the proper protocols when performing measurements using a digital multimeter and practice various methodologies in circuit analysis, which include *Kirchhoff's Voltage and Current Laws* & the *Current and Voltage Divider Rules* (which are derivative of Ohm's Law).

Theory

Current is described as the flow of electric charge and may be represented as the amount of charge flowing through a cross sectional area of wire (**Q**) per unit of time (**t**). Conventionally, the direction of current is the direction in which positive charges move, while negative charges (*carriers*) flow in the opposite direction. In a circuit consisting of an ideal battery and resistor, one can study the relationship between voltage,current, and resistance. When a potential difference is provided, such as an ideal battery or power supply, charges flow within the circuit and its components. To measure the potential difference between any two points within the circuit one can use a voltmeter (such as the potential difference across a resistor or in parallel with said resistor). Meanwhile, ammeters are used to measure the current and must be placed in series with respect the component (whose characteristics are being measured). In 1827, German scientist, Georg Simon Ohm discovered the relationship between the three properties; resistance,current, and *applied* voltage. This empirical relationship is known as Ohm's Law., which states that the current flowing through a resistor has direct proportionality with respect to the *applied* voltage and an inverse proportionality with respect to the impedance or resistance of the resistor.

Gustav Kirchhoff, a German physicist, developed techniques to analyze multiloop circuits. This enabled the analysis of many resistive networks that could not be simplified to series-parallel combinations nor direct evaluation with Ohm's law.

Kirchhoff's *Junction Rule* states that the (algebraic) sum of the current entering a junction must equate to zero. The *Junction Rule* is founded on the basis of the conservation of electric charge. In other words, the charges entering a junction must equate to the total charges leaving, said junction. Kirchhoff's *Loop Rule* states that the (algebraic) sum of the potential differences (voltages) in any given loop, (inclusive of the electromotive forces and resistive electrical components, must equate to zero. The *Loop Rule* is founded on the basis of the conservation energy within a circuit. As a charge moves along a loop the sum of the potential rises is equivalent to the sum of the potential drops.

Materials

Resistors	External Variable DC Power Supply	Digital Multimeter
1kΩ	**12 Volts**	**Breadboard**
2 kΩ	**16 Volts**	**Jumper Wires**
2.2 kΩ	**20 Volts**	
3.3 kΩ	**24 Volts**	
4 kΩ		
4.7 kΩ		

Procedure

In this laboratory exercise we constructed the circuits shown below and collected measurements in regards to the voltages and currents with respect to the resistive components within each. We then, compared these values to the theoretical values, which we deduced using circuit analysis- with the application of Ohm's Law and Kirchhoff's Rules.

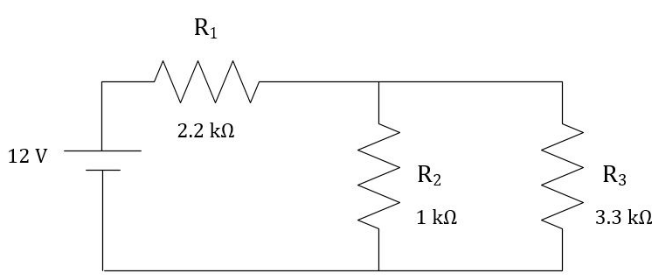


Figure 001

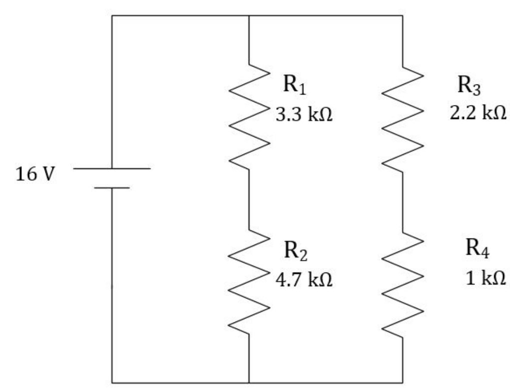


Figure 002

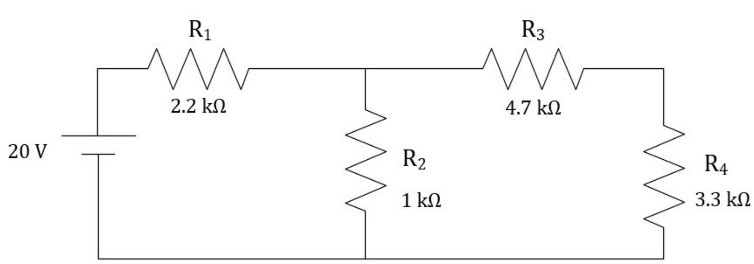


Figure 003

Results

Circuit 001

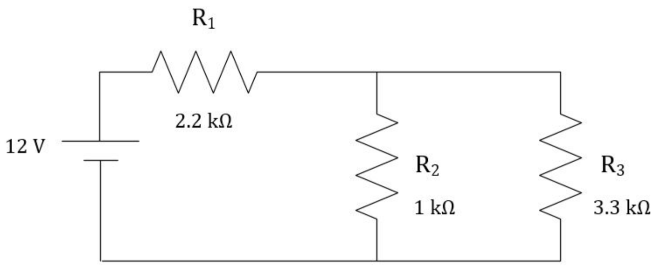
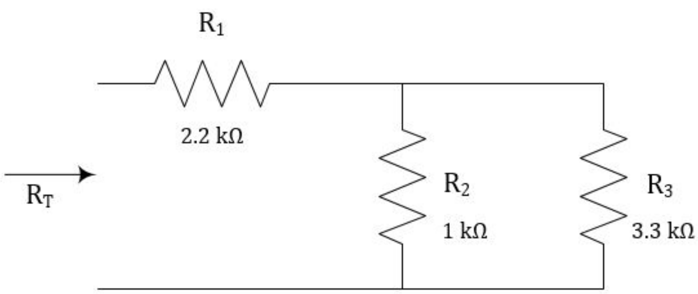


Figure 001

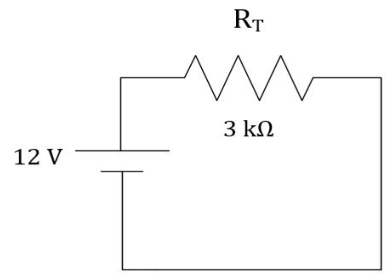


Calculating the Theoretical Total Resistance (R_T)

$$R_2 \| R_3 \;\therefore\; R_{2,3} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_2 R_3}{R_2 + R_3} = \frac{1\text{ k}\Omega \times 3.3\text{ k}\Omega}{1\text{ k}\Omega + 3.3\text{ k}\Omega} \cong 767.4\ \Omega$$

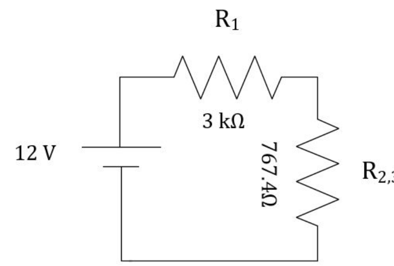
R_1 is in series with $R_{2,3}$ $\therefore$ $R_T = R_1 + R_{2,3} = 2.2\text{ k}\Omega + 767.4\ \Omega = 2967.4\ \Omega \cong 3\text{k}\Omega$

Theoretical Values	Measured Values	Percent Difference
R_1 theoretical = 2.20 kΩ	R_1 measured = 2.177kΩ	1.05%
R_2 theoretical = 1.00 kΩ	R_2 measured = 1.001kΩ	.1%
R_3 theoretical = 3.30 kΩ	R_3 measured = 3.257kΩ	1.3%
R_T theoretical = 2.967kΩ	R_T measured = 2.930kΩ	1.25%



$$I_s = \frac{V_T}{R_T} = \frac{12\text{ V}}{3\text{ k}\Omega} = 4\text{mA}$$

*Note: I_T and I_S are used interchangeably

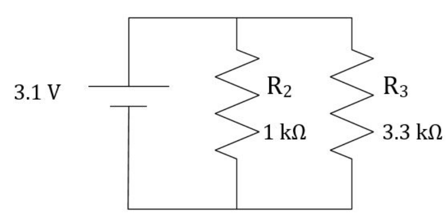


Given $I_{R_1} == I_{R_{2,3}}$

We may use the Voltage Divider Rule

$$V_{R_1} = V_T\left(\frac{R_1}{R_T}\right) = 12\text{V}\left(\frac{2.2\text{k}\Omega}{2967.4\Omega}\right) = 8.9\text{V}$$

$$V_{R_{2,3}} = V_T\left(\frac{R_{2,3}}{R_T}\right) = 12\text{V}\left(\frac{767.4\Omega}{2967.4\Omega}\right) = 3.1\text{V}$$



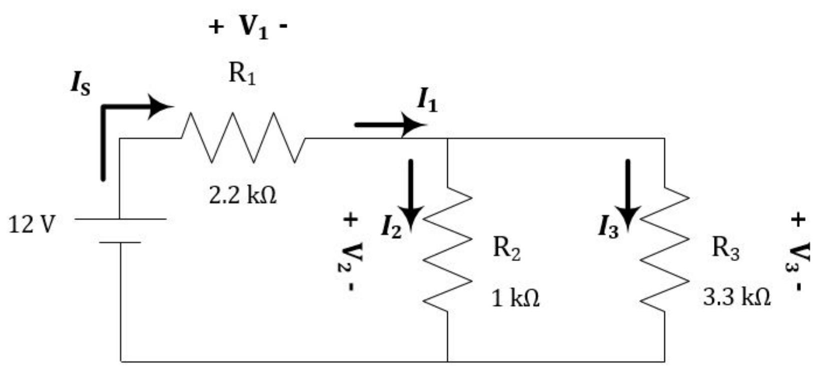
Given $V_{R_2} == V_{R_3}$ & $I_1 = 4\text{mA}$

We may use the Current Divider Rule

$$I_{R_2} = I_T\left(\frac{R_{2,3}}{R_2}\right) = 4\text{mA}\left(\frac{767.4\Omega}{1\text{k}\Omega}\right) = 3\text{mA}$$

$$I_{R_3} = I_T\left(\frac{R_{2,3}}{R_3}\right) = 4\text{mA}\left(\frac{767.4\Omega}{3.3\text{k}\Omega}\right) = 1\text{mA}$$

Tabular Results for Circuit 001



Circuit 001

Current		Calculated	Measured	% Difference
I_s		4 mA	4.1 mA	2.5
I_1		4 mA	4.1 mA	2.5
I_2		3 mA	3.09 mA	3.0
I_3		1 mA	.95 mA	5.0

Voltage		Calculated	Measured	% Difference
V_1		8.9 V	8.78 V	1.35
V_2		3.10 V	3.10 V	0 %
V_3		3.10 V	3.10 V	0%

➢ How is are the voltages V_2 and V_3 related? Why?

The voltages,V_2 & V_3 , are equivalent because the components are placed in parallel with respect to one another.

➢ Referring to the circuit above and the table, does $E = V_1 + V_2$ as required by Kirchhoff's Law?

The measured and calculated values for the voltages V_1 & V_2 fulfill Kirchhoff's Loop Rule (Kirchhoff's Voltage Law)

Circuit 002

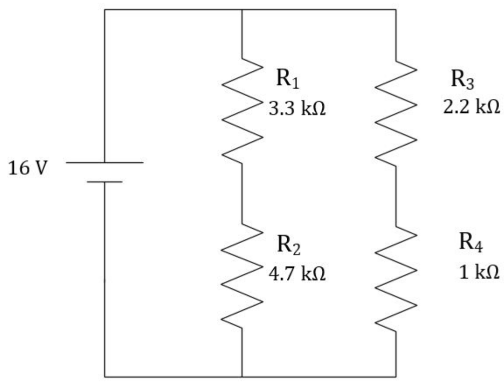
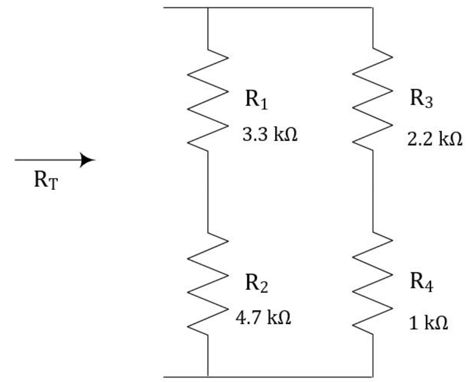


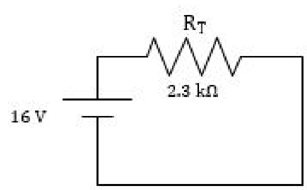
Figure 002



Calculating the Theoretical Total Resistance (R_T)

$$R_T = \frac{1}{\frac{1}{R_1 + R_2} + \frac{1}{R_3 R_4}} = \frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2 + R_3 + R_4)} = \frac{(3.3\text{ k}\Omega + 4.7\text{ k}\Omega)(2.2\text{ k}\Omega + 1\text{ k}\Omega)}{(3.3\text{ k}\Omega + 4.7\text{ k}\Omega + 2.2\text{ k}\Omega + 1\text{ k}\Omega)}$$

$$= \frac{(8\text{ k}\Omega)(3.2\text{ k}\Omega)}{(8\text{ k}\Omega + 3.2\text{ k}\Omega)} = \frac{25{,}600{,}00\ \Omega}{11.2\text{ k}\Omega} = 2{,}285\ \Omega = 2.3\text{ k}\Omega$$

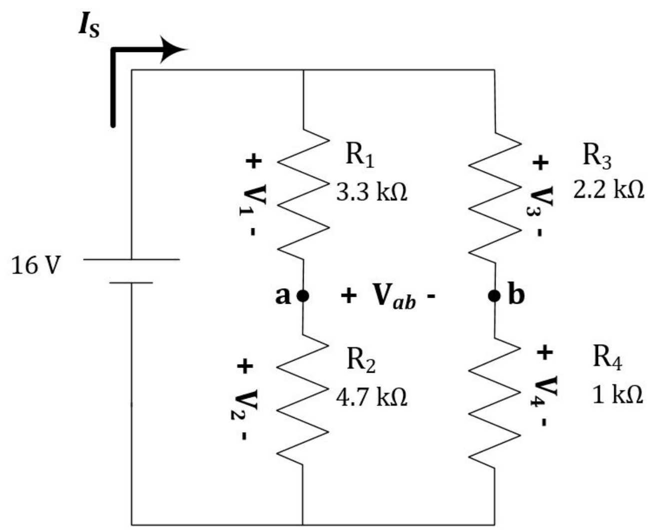


$$R_T = \frac{1}{\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}} = \frac{1}{\frac{1}{3.3\text{K}\Omega + 4.74\text{K}\Omega} + \frac{1}{2.2\text{K}\Omega + 1\text{K}\Omega}} = 2.285K\Omega$$

$$R_T = 2.3K\Omega$$

Theoretical Values	Measured Values	Percent Difference
R_1 theoretical = 3.30 kΩ	R_1 measured = 3.257kΩ	1.3%
R_2 theoretical = 4.70 kΩ	R_2 measured = 4.74kΩ	0.9%
R_3 theoretical = 2.20 kΩ	R_3 measured = 2.177kΩ	1.0%
R_4 theoretical = 1.00 kΩ	R_T measured = 1.001kΩ	0.1%

Tabular Results for Circuit 002



Circuit 002

		Calculated	Measured	% Difference
V_2		9.4V	9.50V	1.06
V_3		4.98V	5.40V	8.44
V_{ab}		4.36V	4.50V	3.21
I_S		6.98A	.6.95A	0.43

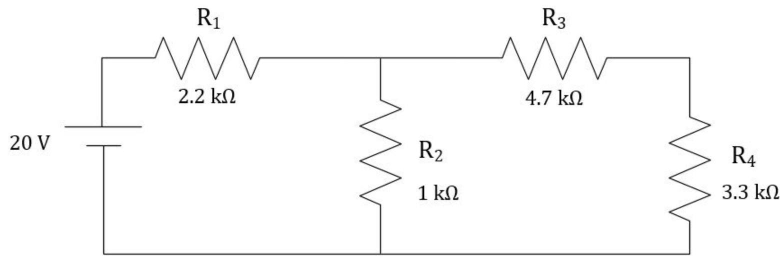
- How is the total voltage across the two series elements $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ related to the applied voltage $\boldsymbol{E}$? Why?

The sum of the voltages across $\mathbf{V}_{R1}$ and $\mathbf{V}_{R2}$ are equivalent to the electrical potential of the 16 V denoted as **E** *,because the resistors* $\mathbf{R}_1$ *and* $\mathbf{R}_2$ with respect to the *power source* are placed in parallel. Additionally, if one was to apply Kirchhoff's Loop Rule (also known as Kirchhoff's Voltage Law), the 16 V power source and resistors $\mathbf{R}_1$ and $\mathbf{R}_2$ complete one loop. Thus, the sum of the voltage rises and voltage drops must equate to zero. In essence, the sum of the voltage drops equate to the voltage rises ($\boldsymbol{E}$= $\mathbf{V}_{R1}$ + $\mathbf{V}_{R2}$).

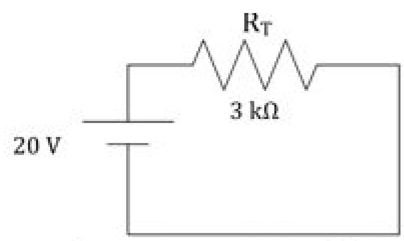
- How is the total voltage across the two series elements R_3 and R_4 related to the applied voltage E ? Why?

The sum of the voltages across $\mathbf{V}_{R3}$ and $\mathbf{V}_{R4}$ are equivalent to the electrical potential of the 16 V denoted as **E** *,because the resistors* $\mathbf{R}_3$ *and* $\mathbf{R}_4$ with respect to the *power source* are placed in parallel.Additionally, if one was to apply Kirchhoff's Loop Rule (also known as Kirchhoff's Voltage Law), the 16 V power source and resistors $\mathbf{R}_3$ and $\mathbf{R}_4$ complete one loop. Thus, the sum of the voltage rises and voltage drops must equate to zero. In essence, the sum of the voltage drops equate to the voltage rises ($\boldsymbol{E}$= $\mathbf{V}_{R3}$ + $\mathbf{V}_{R4}$).

Circuit 003



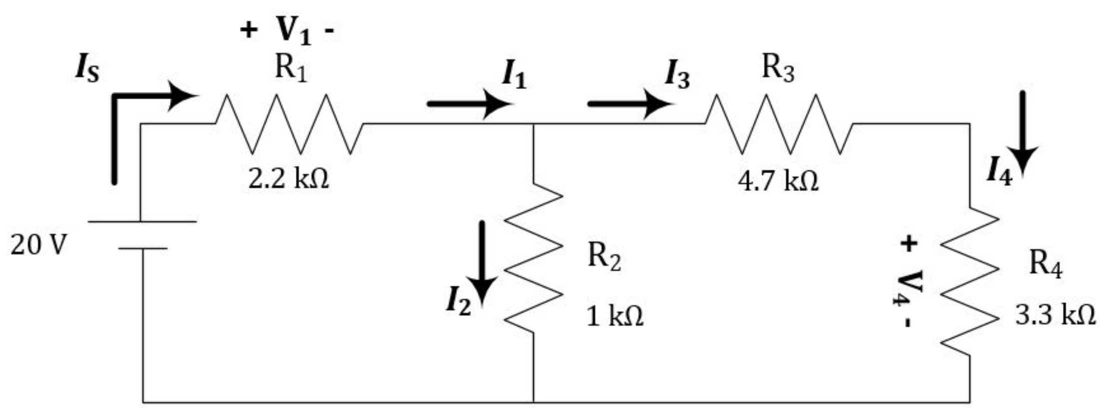
Circuit 003



$$R_T = R_1 + \frac{R_2 \times (R_3 + R_4)}{R_2 + R_3 + R_4} = 2.2\text{K}\Omega + \frac{1\text{K}\Omega \times (4.7\text{K}\Omega + 3.3\text{K}\Omega)}{1\text{K}\Omega + 4.7\text{K}\Omega + 3.3\text{K}\Omega} = 3.08K\Omega$$

$$R_T = 3K\Omega$$

Tabular Results for Circuit 003



Circuit 003

		Calculated	Measured	% Difference
V_4		2.112V	2.37V	21.68
I_S		6.45mA	6.5mA	0.78
R_T		3.1K	3.206K	3.42

Conclusion

Throughout these exercises, we have verified multiple principles in regards to circuits structured with components in series, parallel, and series-parallel positions. When components are in series they share the same current and the voltage is shared in ratio to the resistance. The total resistance of a circuit containing strictly resistors in series formation is equivalent to the sum of all resistances of each resistor. To calculate the voltage of each component one may use the voltage divider rule.

When components are in parallel to one another they share the same voltage and the current is shared proportionally with respect to the resistance. The total resistance of a circuit containing resistors placed parallel to one another is equivalent of the inverse of the inverse sum of all resistances of each resistor. To calculate the current flowing through each resistive component one can use the current divider rule. In addition, when adding additional resistors parallel to a given circuit the overall resistance is decreased.