Linear dielectrics

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A linear dielectric is a material in which the polarization vector is proportional to the electric field. Many materials behave as linear dielectrics as long as the field is not too strong.

$$\boxed{\bar{P} \equiv \varepsilon_0 \chi_e \bar{E}}$$

χ_e = electric susceptibility (dimensionless quantity)

It is important to point out that in the equation above E is the total electric field; the equation above cannot be used to calculate P from the external electric field in which one places a piece of dielectric material.

The electric displacement in a linear dielectric assumes a simple form

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} = \varepsilon_0 \bar{E} + \varepsilon_0 \chi_e \bar{E} = \underbrace{\varepsilon_0 (1 + \chi_e)}_{\equiv \varepsilon} \bar{E}$$

$$\boxed{\varepsilon \equiv \varepsilon_0 (1 + \chi_e)}$$

ELECTRIC PERMITTIVITY

$$\boxed{\bar{D} = \varepsilon \bar{E}}$$

in a linear dielectric

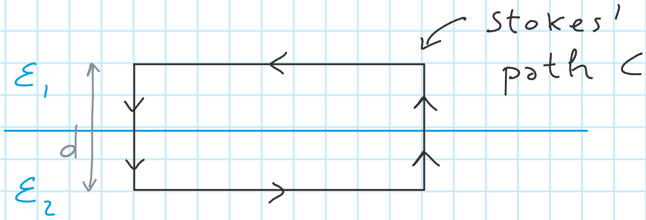
$$[\varepsilon] = [\varepsilon_0] = \frac{C^2}{N m^2}$$

Sometimes it is useful to introduce the relative permittivity or dielectric constant of a material

$$\boxed{\varepsilon_r \equiv 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}}$$

observe that $\varepsilon_r > 1$

For a linear material then one is tempted to conclude that the curl of D is zero everywhere, since D is proportional to E. However, one should be particularly careful in analyzing this situation. In fact, let's apply Stokes' theorem to a path that crosses the boundary between two dielectrics.



$$\oint_C \bar{E} \cdot d\bar{\ell} = 0$$

$$E_1^{\parallel} = E_2^{\parallel}$$

however, in general

$$D_1^{\parallel} = \varepsilon_1 E_1^{\parallel} \neq D_2^{\parallel} = \varepsilon_2 E_2^{\parallel}$$

$$\downarrow$$

$$\nabla \times \bar{D} \neq 0 \text{ at the boundary}$$

The curl of D must be different from 0 at the boundary because one can take the limit d --> 0 without affecting the argument above.

Of course the argument above does not stand when the space under consideration is filled with only one type of dielectric. In that case

$$\nabla \cdot \bar{D} = \rho_f \quad ; \quad \nabla \times \bar{D} = 0$$

If the free charge was placed it vacuum, it would create a field that satisfies the equation

$$\nabla \cdot \bar{E}_{vac.} = \frac{\rho_f}{\varepsilon_0}$$

But then $\quad$ but $\nabla\cdot\bar{D} = \rho_f$

$$\nabla\cdot\left(\varepsilon_0 \bar{E}_{vac}\right) = \rho_f \longrightarrow \boxed{\bar{D} = \varepsilon_0 \bar{E}_{vac}} \qquad (I)$$

One can then put (I) together with the relation between D and the full field E in a linear dielectric:

$$\bar{D} = \varepsilon \bar{E} \rightarrow \bar{E} = \frac{1}{\varepsilon}\bar{D} = \frac{\varepsilon_0}{\varepsilon}\bar{E}_{vac} = \frac{1}{\varepsilon_r}\bar{E}_{vac}$$

When all space is filled with a homogenous linear dielectric, the field everywhere differs from the field one would have in empty space simply by a factor. Since the dielectric constant is a number > 1, the electric field in the homogeneous linear dielectric is smaller than the field that the same free charge would create in vacuum. In this case, one can then write E in terms of the electric potential that the free charges would create in vacuum

$$\left.\begin{aligned} &\bar{E}_{vac} = -\nabla\varphi_{vac} \qquad \bar{E} = -\nabla\varphi \\ &\bar{E} = \frac{1}{\varepsilon_r}\bar{E}_{vac} = -\frac{1}{\varepsilon_r}\nabla\varphi_{vac} \end{aligned}\right\} \quad \varphi = \frac{1}{\varepsilon_r}\varphi_{vac}$$

For example, for a free point charge q in a dielectric

$$\varphi = \frac{1}{\varepsilon_r}\frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon}\frac{q}{r}$$