Magnetic forces

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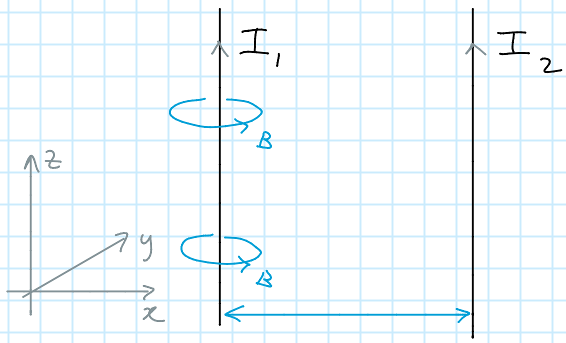
A current produces a magnetic field in the space which surrounds it. If a moving charge or a current is placed in this field, the magnetic field will apply a force on this charge or current. This force is the Lorentz force

$$\bar{F} = q\bar{v} \times \bar{B}$$

Here we want to analyze the force that one current applies on another current through this mechanism.

Simplest case: Force between two straight, parallel wires carrying currents



The charges responsible for the current I_2 will feel a force

$$\bar{F} = q\bar{v} \times \bar{B} = q\bar{v} \times \underbrace{\left(\frac{\mu_0 I_1}{2\pi d}\right)}_{\bar{B} \text{ produced by } I_1} \hat{j}$$

If the wire carrying the current I_2 has a cross section A one can write

$$I_2 = j_2 A = n q v A \qquad qv = \frac{I_2}{nA}$$

(n: # charges per unit volume)

$$\bar{F} = \frac{I_2}{nA} \frac{\mu_0 I_1}{2\pi d} \hat{k} \times \hat{j} = -\frac{1}{nA} \frac{\mu_0 I_1 I_2}{2\pi d} \hat{i}$$

(nA: # charges per unit length)

The force per unit length is then

$$\bar{f} = nA\bar{F} = -\frac{\mu_0 I_1 I_2}{2\pi d}\hat{i}$$

The force is attractive if I_1 and I_2 point in the same direction, repulsive if they point in opposite directions.

Force between generic currents

If a current I_1 is confined on a wire that follows the curve C_1,the magnetic field produced by that current can be written as

$$\bar{B}(\bar{r}) = \frac{\mu_0 I_1}{4\pi}\oint_{C_1} \frac{d\bar{r}' \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

The force applied by this field on a second current density j_2 is then

$$\bar{F} = \int d^3r \; \bar{J}_2(\bar{r}) \times \bar{B}(\bar{r})$$

If the second current I_2 runs in a wire which follows a path C_2 the force becomes

$$\bar{F} = \frac{\mu_0 I_1 I_2}{4\pi}\oint_{C_1}\oint_{C_2} d\bar{r} \times \left(d\bar{r}' \times \frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}\right)$$

The integral above is usually quite difficult to evaluate.