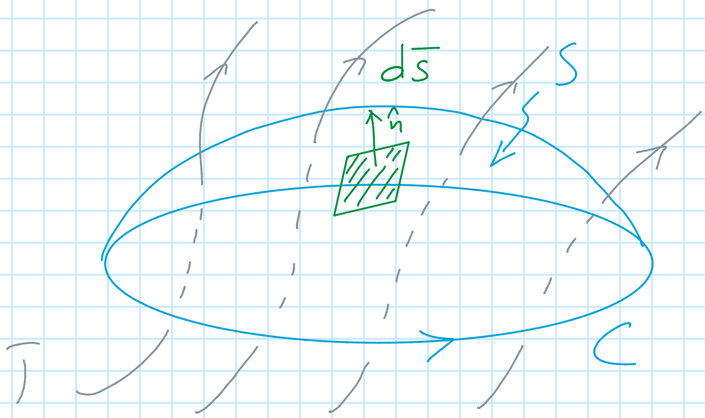
Ampere's Law

Sunday, January 13, 2019 4:20 PM

$$\boxed{\nabla \times \bar{B} = \mu_0 \bar{J}}$$

AMPERE'S LAW

We already saw how to rewrite Ampere's law in integral form by using Stoke's theorem

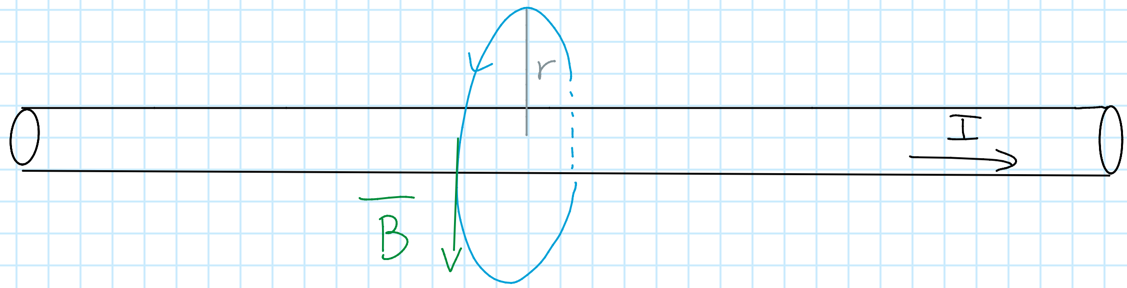


$$\int_S \nabla \times \bar{B} \cdot d\bar{s} = \oint_C \bar{B} \cdot d\bar{\ell} = \mu_0 \underbrace{\int_{S'} \bar{J} \cdot d\bar{s}}_{\text{current flowing through } S'} = \mu_0 I$$

In most cases Ampere's law alone is not sufficient to determine the magnetic field. However, in situations characterized by an obvious symmetry, it is sufficient to use the integral form of Ampere's law to figure out what the magnetic field is. In those cases, the symmetry of the problem dictates a magnetic field that obviously satisfies also Gauss' law for the magnetic field.

Example 1: Long straight wire

A long straight wire has a cylindrical symmetry, therefore also the magnetic field has to respect that symmetry:



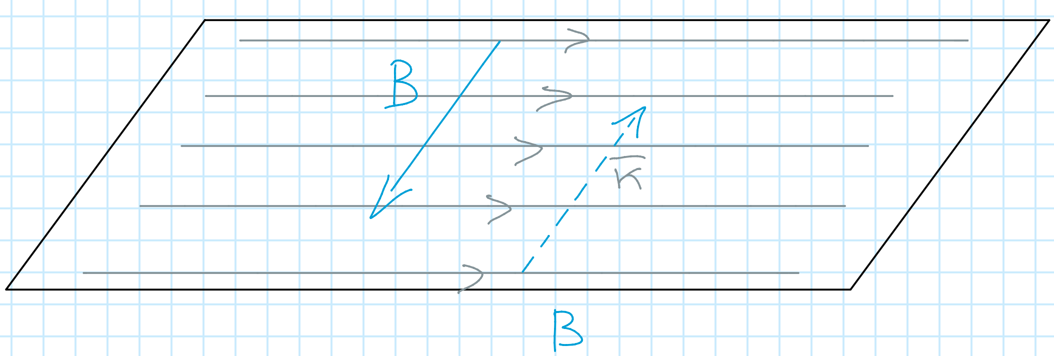
The magnitude of B can only depend on r

$$\oint_C \bar{B}\cdot d\bar{\ell} = B(r)\int_0^{2\pi} r\, d\varphi = 2\pi r\, B(r) = \mu_0 I$$

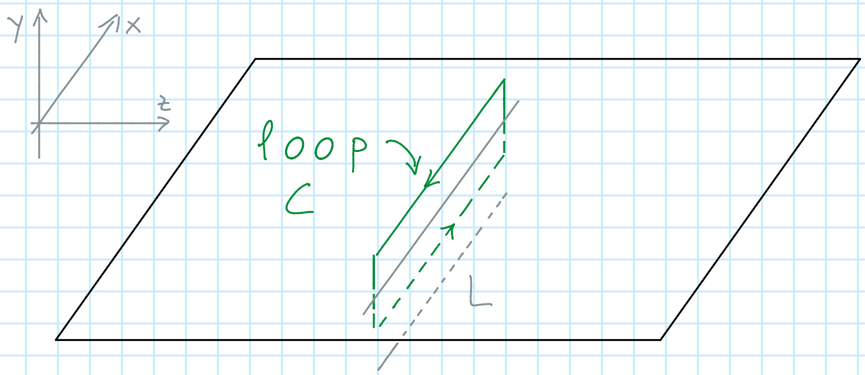
$$\boxed{\bar{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\varphi}}$$

Example 2: Surface current

Consider the case in which one lies downs many straight long wires parallel to each other. In this case one obtains a surface charge density K. (The dimensions of K are current over length.) By considering the magnetic field due to a current in a long straight wire one see that in this case the field will be parallel to the surface and perpendicular to K.



One can then apply Ampere's law to a loop of the form



$$\oint_C \overline{B}\cdot d\overline{\ell} = 2LB = \mu_0 K L \qquad B = \frac{\mu_0 K}{2}$$

One can then see that

$$\overline{B}(y\to 0^+) = -B\,\hat{\imath} = -\frac{\mu_0 K}{2}\,\hat{\imath}$$

$$\overline{B}(y\to 0^-) = B\,\hat{\imath} = \frac{\mu_0 K}{2}\,\hat{\imath}$$

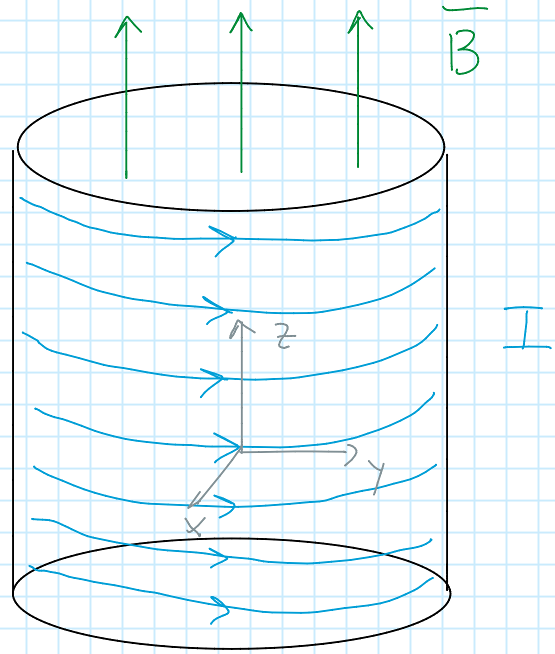
$$\overline{B}(y\to 0^+) - \overline{B}(y\to 0^-) = -\mu_0 K\,\hat{\imath}$$

The above is in agreement with the discontinuity of the component of the magnetic field parallel to a surface carrying a surface current, which we discussed in general terms previously.

Remember that for an electric field in presence of a surface charge, the component of the field perpendicular to the surface is discontinuous while the component of the field parallel to the surface is continuous. For a magnetic field in presence of a surface current it is the component of the field parallel to the surface which is discontinuous and the component of the field perpendicular to the surface which is continuous.

Example 3: Solenoid

A solenoid is a wire tightly wrapped around a cylinder. We consider the standard case in which the cross section of the cylinder is a circle.



The cylindrical symmetry of the problem dictates that the magnetic field is directed along the axis of the cylinder, which we choose to be the z axis of our frame of reference. We use cylindrical coordinates. Because of symmetry, the magnitude of the magnetic field can only depend on the radial coordinate.

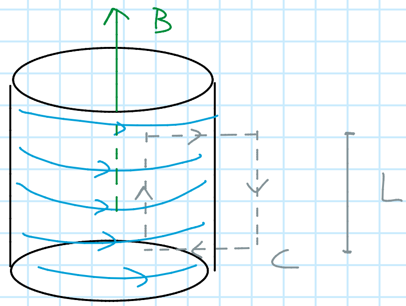
$$\overline{B} = B(\rho)\,\hat{k}$$

Ampere's law in differential form requires that

$$\nabla \times \overline{B} = 0 \quad \longrightarrow \quad -\frac{\partial B}{\partial \rho}\,\hat{\varphi} = 0$$

$$\rightarrow B(\rho) = \text{const.}$$

Since B should vanish infinitely far away from the solenoid and the derivative of B with respect to rho is 0, then B = 0 outside the solenoid. In order to determine B inside the solenoid we apply Ampere's law in integral form to the Amperian circuit below



$$\oint \bar{B} \cdot d\bar{\ell} = BL = \mu_0 I N L$$

number of windings per unit length

$$\boxed{\bar{B} = \mu_0 I N \hat{k}}$$

inside the solenoid