Divergence 2

Tuesday, February 1, 2022 11:26 AM

G problem 16

Calculate the divergence of the following vector

$$\bar{v} = \frac{\hat{r}}{r^2} = \frac{\bar{r}}{r^3} \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$\bar{v} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$\nabla \cdot \bar{v} = \sum_{i=1}^{3} \frac{1}{(\quad)^{\frac{3}{2}}} - \frac{3}{2} \frac{2x_i^2}{(\quad)^{\frac{5}{2}}}$$

$$= \frac{1}{r^3}\left(1 - \frac{3x^2}{r^2} + 1 - \frac{3y^2}{r^2} + 1 - \frac{3z^2}{r^2}\right)$$

$$= \frac{3}{r^5}\left(r^2 - x^2 - y^2 - z^2\right) = 0$$