Double cross product

Thursday, January 27, 2022 9:40 AM

G problem 6 page 8

Prove that

$$\bar{A}\times(\bar{B}\times\bar{C}) + \bar{B}\times(\bar{C}\times\bar{A}) + \bar{C}\times(\bar{A}\times\bar{B}) \doteq 0$$

$$\varepsilon_{ijk} A_j(\varepsilon_{k\ell m} B_\ell C_m) + \varepsilon_{ijk} B_j(\varepsilon_{k\ell m} C_\ell A_m) + \varepsilon_{ijk} C_j(\varepsilon_{k\ell m} A_\ell B_m) \overset{?}{=} 0$$

$$\varepsilon_{ijk}\varepsilon_{k\ell m} = +\varepsilon_{kij}\varepsilon_{k\ell m} = \delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}$$

$$(\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell})(A_j B_\ell C_m + B_j C_\ell A_m + C_j A_\ell B_m) =$$

$$\cancel{B_i A_j C_j} - \cancel{C_i A_j B_j} + \cancel{C_i A_j B_j} - A_i \cancel{B_j C_j}$$
$$+ A_i \cancel{B_j C_j} - \cancel{B_i A_j C_j} = 0 \quad \checkmark$$

Under which condition does

$$\bar{A}\times(\bar{B}\times\bar{C}) = (\bar{A}\times\bar{B})\times\bar{C} \quad ?$$

$$\varepsilon_{ijk} A_j \varepsilon_{k\ell m} B_\ell C_m = \varepsilon_{ijk}(\varepsilon_{j\ell m} A_\ell B_m) C_k$$
$$(\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}) A_j B_\ell C_m = (-\delta_{i\ell}\delta_{km} + \delta_{im}\delta_{k\ell}) A_\ell B_m C_k$$

$$B_i A_j C_j - C_i A_j B_j = B_i A_k C_k - A_i B_k C_k$$

$$\cancel{B_i \bar{A}\cdot\bar{C}} - C_i \bar{A}\cdot\bar{B} = \cancel{B_i \bar{A}\cdot\bar{C}} - A_i \bar{B}\cdot\bar{C}$$

$$\cancel{+} C_i \bar{A}\cdot\bar{B} = \cancel{+} A_i \bar{B}\cdot\bar{C}$$

The equality above is verified if

$$C_i = \alpha A_i \qquad \bar{A} \parallel \bar{C}$$