Example: Line segment

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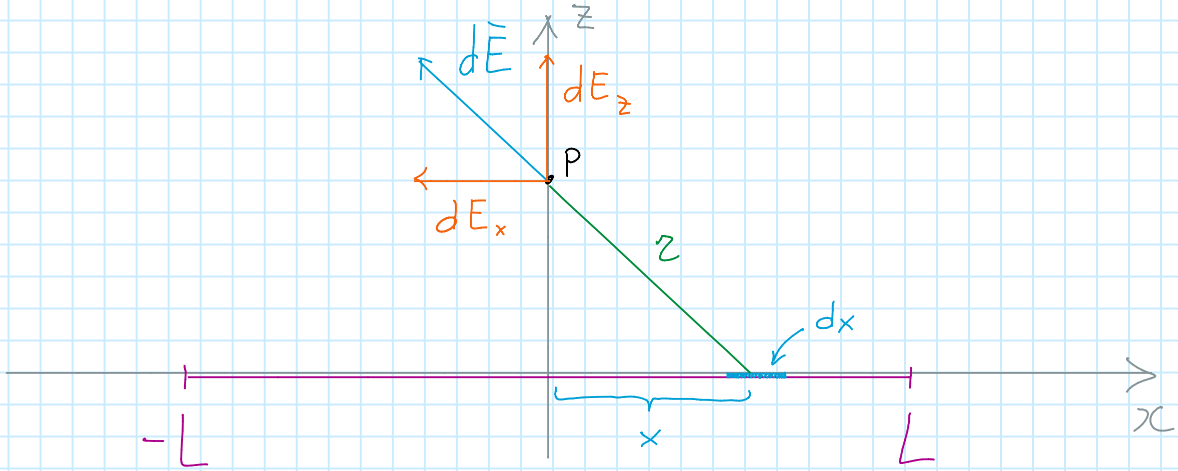
Find the electric field a distance z above the midpoint of a straight line segment of length 2L that carries a uniform line charge λ



The horizontal components of the electric field cancel out upon integration because of the symmetry of the segment with respect to the x axis. The total E is pointing in the direction of the z axis.
However, one can also see this directly from the calculation.

$$\bar{r} = z\,\hat{k} \qquad \bar{r}' = x\,\hat{\imath} \qquad dl' = dx$$

$$\bar{\mathscr{r}} = \bar{r} - \bar{r}' = z\,\hat{k} - x\,\hat{\imath} \qquad \mathscr{r} = \sqrt{x^2 + z^2}$$

$$\hat{\mathscr{r}} = \frac{\bar{\mathscr{r}}}{\mathscr{r}} = \frac{z\,\hat{k} - x\,\hat{\imath}}{\sqrt{x^2+z^2}}$$

$$\bar{E}(z) = \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{\lambda}{x^2+z^2} \frac{z\,\hat{k} - x\,\hat{\imath}}{\sqrt{x^2+z^2}}\, dx$$

$$= \frac{1}{4\pi\varepsilon_0}\left[z\,\hat{k} \int_{-L}^{L} \frac{\lambda}{(x^2+z^2)^{\frac{3}{2}}}\,dx - \hat{\imath} \int_{-L}^{L} \frac{x}{(x^2+z^2)^{\frac{3}{2}}}\,dx \right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[z\,\hat{k} \left(\frac{x}{z^2\sqrt{z^2+x^2}} \right)\Bigg|_{-L}^{L} - \hat{\imath} \left(-\frac{1}{\sqrt{z^2+x^2}} \right)\Bigg|_{-L}^{L} \right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \frac{2L}{z\sqrt{z^2+L^2}} \hat{k}$$

It is useful to consider a two limits:

if $z >> L$ $\qquad E \cong \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{z^2}$ $\qquad$ same as a point charge

if $z << L$ $\qquad E \cong \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{z}$ $\qquad$ same as an infinite straight wire

Problem: Find the electric field a distance z above the center of a flat circular loop of radius r that carries a λ

Problem: Find the electric field a distance z above the center of a flat circular disk of radius R that carries a uniform surface charge σ. What does your formula give in the limit R --> infinity? And in the case z >> R ?