Coulomb's law

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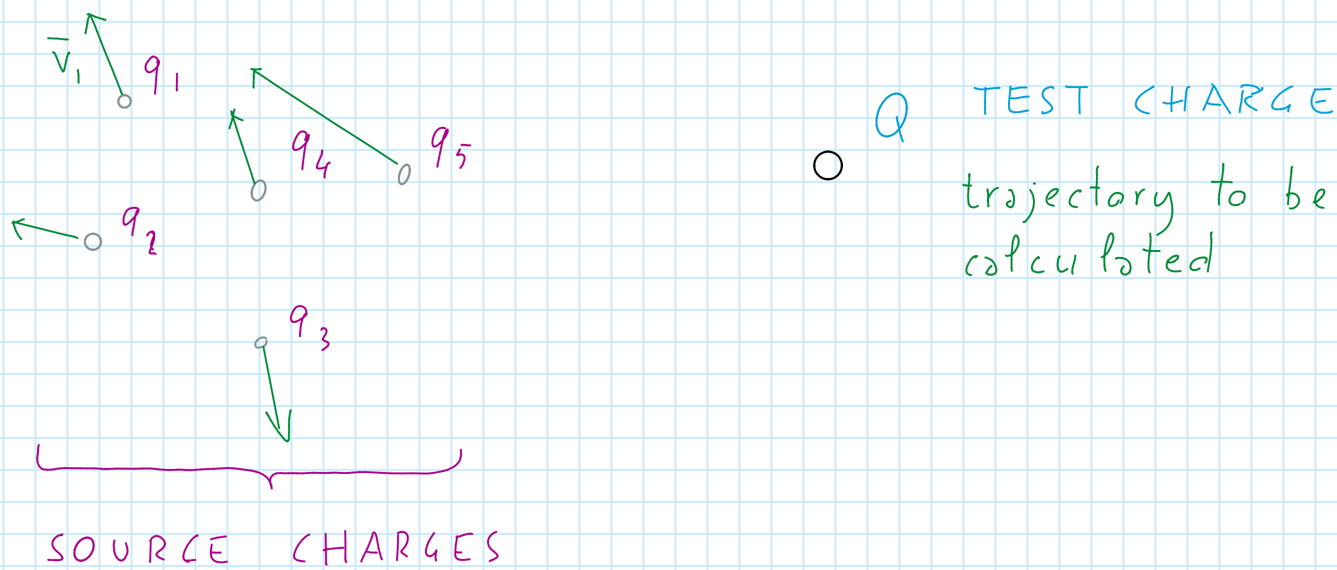
Griffith ch 1

Electrodynamics should allow us to answer the following question:
Given the location and velocities of an arbitrary number of charges as a function of time, can one predict what is the force acting on a test charge Q?



Principle of superposition:
The interaction between any two charges is completely unaffected by the presence of the other charges.

Therefore if one knows the force applied by each single charge on the test charge, the net force on the test charge is just the sum of the charges applied by each one of the source charges on it

$\bar{F}_1$ force applied by q_1 on Q
$\bar{F}_2$ " by q_2 on Q
$\vdots$
$\bar{F}_n$ " by q_n on Q

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \ldots + \bar{F}_n$$

It seems easy to solve this problem, but life is complicated by the simultaneous presence of electric and magnetic fields, that change in time if the sources are moving, by the fact that changes in the field travel at finite speed and by the fact that we often deal with practically continuous charge distributions.

Therefore is convenient to start by considering the case in which the source charges are stationary (but the test charges can be moving). This is what is referred to as Electrostatics.

ME for electrostatics

We already know that everything in electricity and magnetism is described by ME. Therefore, also the equations that we need to solve in the case of electrostatics should be included in ME.

If the source charges are not moving, the current density is zero. If the current density is zero, there is no magnetic field. If the source charges are stationary, the electric field that they produce cannot depend on time, therefore

$$\boxed{\nabla \cdot \bar{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \bar{E} = 0}$$

ELECTROSTATICS EQUATION

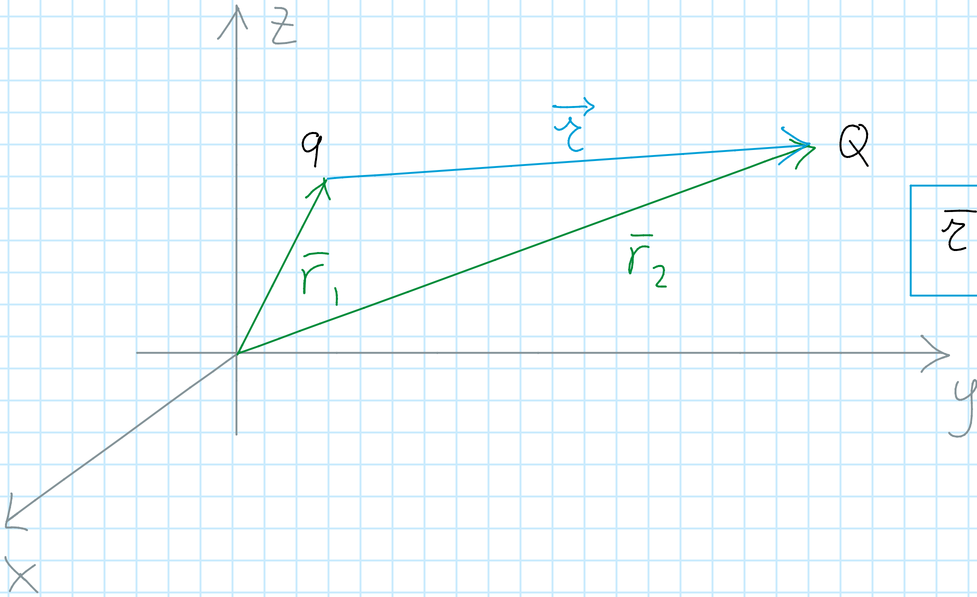
Coulomb's law

As we know from general physics, Gauss' law for E is completely equivalent to Coulombs law, that says that the force between two point charges is

$$\boxed{\bar{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r}}$$

The force can be attractive or repulsive, depending on the sign of the charges.
The permittivity of free space is

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N\ m^2}$$



$$\bar{\mathscr{r}} \equiv \bar{r}_2 - \bar{r}_1$$

Electric field

If there are several source charges, the total force acting on the test charge is

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \dots + \bar{F}_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{\mathscr{r}_1^{\,2}}\hat{\mathscr{r}}_1 + \frac{q_2 Q}{\mathscr{r}_2^{\,2}}\hat{\mathscr{r}}_2 + \frac{q_3 Q}{\mathscr{r}_3^{\,2}}\hat{\mathscr{r}}_3 + \dots + \frac{q_n Q}{\mathscr{r}_n^{2}}\hat{\mathscr{r}}_n\right)$$

But we also know that the relation between the total force on the test charge and the electric field in the spot where the test charge is located is

$$\boxed{\bar{F} = Q\,\bar{E}}$$

Consequently

$$\boxed{\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^{2}}\hat{\mathscr{r}}_i}$$

Formally, this is the end of the story. However, most of the time one needs to deal with continuous charge distributions.

Problem: Find the electric field (magnitude and direction) a distance z above the midpoint between equal and opposite charges ($\pm q$), a distance d apart.