Continuous charge distributions

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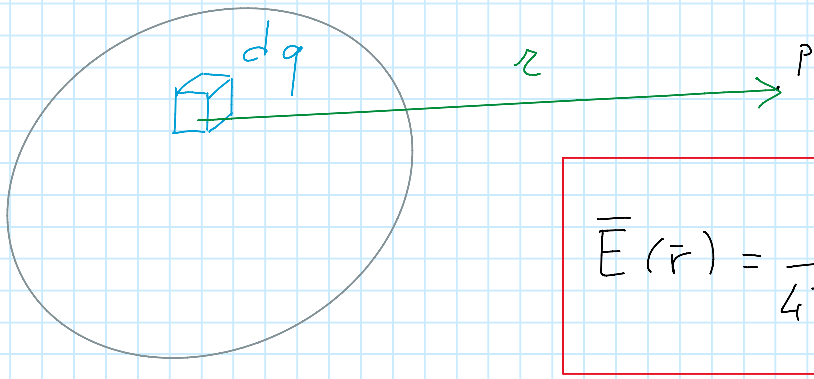
Charge is quantized and any charge is a collection of protons and electrons, that can be modeled as point particles for our purposes. However, the vast majority of macroscopic charges contain so many electrons and protons that for all practical purposes they can be more conveniently modeled as continuous charge distributions.

In order to find the total field E in a given point in space, one must then integrate over the field due to each infinitesimal source charge dq



$$\boxed{\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, dq}$$

There are three types of continuous charge distributions that we can encounter:
Line charges, surface charges, and volume charges

LINE CHARGE

$$dq = \lambda\, d\ell \qquad [\lambda] = \frac{C}{m}$$

$$\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\int_{line} \frac{\lambda(\bar{r}')}{\mathscr{r}^2}\,\hat{\mathscr{r}}\, d\ell'$$

SURFACE CHARGE

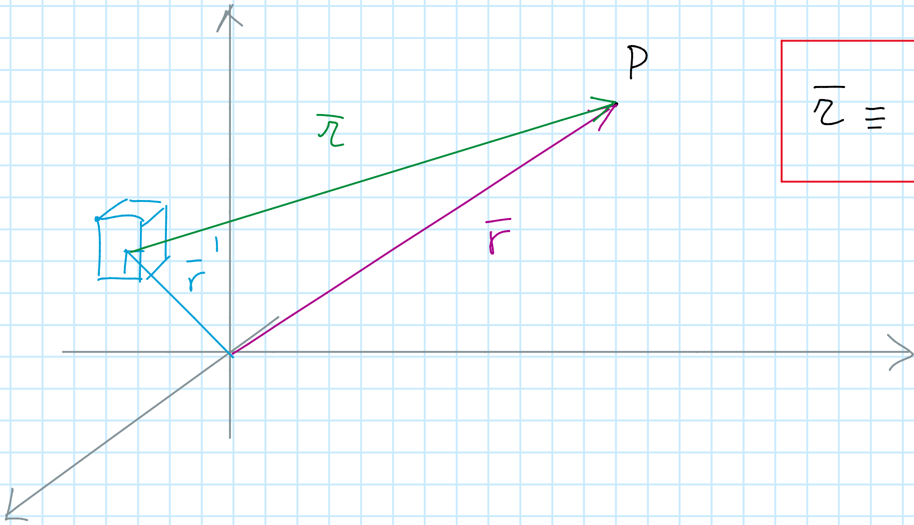
$$dq = \sigma\, dS \qquad [\sigma] = \frac{C}{m^2}$$

$$\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\int_{surface} \frac{\sigma(\bar{r}')}{\mathscr{r}^2}\,\hat{\mathscr{r}}\, dS'$$

VOLUME CHARGE $\quad dq = \rho \, dV \qquad [\rho] = \frac{C}{m^3}$

$$\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \int_{\text{Volume}} \frac{\rho(\bar{r}')}{\mathscr{r}^2} \hat{\mathscr{r}} \, dV'$$

Remember the meaning of $\bar{\mathscr{r}}$



$$\boxed{\bar{\mathscr{r}} \equiv \bar{r} - \bar{r}'}$$