The displacement current

Sunday, May 5, 2019 2:25 PM

Only one term in Maxwell's equation was first discovered through theory rather than experiment, the displacement current, which was proposed by Maxwell

$$\nabla \times \bar{B} = \mu_0 \Big(\bar{J} + \underbrace{\varepsilon_0 \frac{\partial \bar{E}}{\partial t}}_{\text{displacement current}} \Big)$$

Observe that the displacement current does not have the physical dimensions of charge over time, and therefore it is NOT a current, in spite of its name.

Observe that if the displacement current was not present, the equation above would read

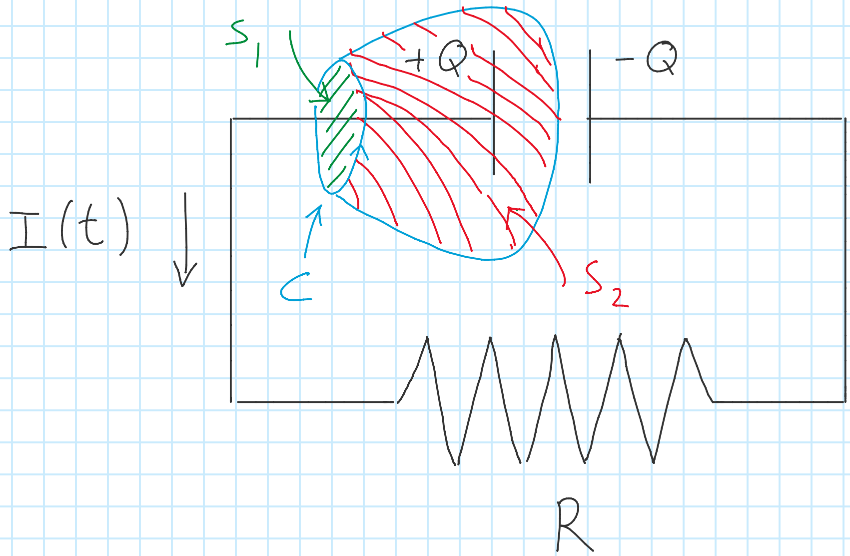
$$\nabla \times \bar{B} = \mu_0 \bar{J}$$

By taking the divergence of the above one would find

$$\mu_0 \nabla \cdot \bar{J} = \nabla \cdot (\nabla \times \bar{B}) = 0$$

By applying Gauss theorem, one could than conclude that the flux of j through any closed surface would have to be zero. This goes against what on observes in nature: If we have a bunch of equal sign charges that are free to move clustered in small volume, they will move away from each other and eventually they will leave the volume. This will correspond to a flux of the current density through the volume surface which is different from zero.

Most books illustrate the need for the displacement current by considering the following ideal situation: A charge capacitor is connected to a resistance. The capacitor discharges creating a time-dependent current (observe that in magnetostatics we discussed only steady currents!)



Now one can apply the integral form of Ampere's law without the displacement current to the surface S_1 bound by the oriented path C. One finds as expected

$$\oint_C \bar{B} \cdot d\bar{\ell} = \mu_0 I$$

However, one is allowed to apply Ampere's circulation law to any surface bound by C, including the surface S_2 that goes between the plates. However, there is no current piercing the surface S_2 so that one would conclude that

$$\oint_C \bar{B} \cdot d\bar{\ell} = 0$$

These two results cannot be both correct. The explanation of this is that the integral form of Ampere's law above holds only for steady currents. Here one should take into account the contribution of the displacement current. The field between the two plates at a given moment in time is

$$E = \frac{V}{d} = \frac{Q}{C} \frac{1}{d} = Q \frac{\cancel{d}}{\varepsilon_0 A} \frac{1}{\cancel{d}} = \frac{Q}{\varepsilon_0 A}$$

capacitance

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0 A} \frac{dQ}{dt} = \frac{1}{\varepsilon_0 A} I(t)$$

Ampere's law with the contribution of the displacement current is

$$\oint_C \bar{B} \cdot d\bar{\ell} = \int_{S_2} (\nabla \times \bar{B}) \cdot d\bar{S} = \int_{S_2} \left(\mu_0 \bar{J} + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t} \right) \cdot d\bar{S}$$

$$= \mu_0 \varepsilon_0 \int_{S_2} \frac{\partial \bar{E}}{\partial t} \cdot d\bar{S}$$

If we consider the right side of S_2 almost flat and the capacitor's plates "large" in comparison to their distance one finds $\bar{E} \parallel d\bar{S}$

$$\oint \bar{B} \cdot d\bar{\ell} = \mu_0 \varepsilon_0 \int_{S_2} \frac{\partial E}{\partial t} dS = \mu_0 \varepsilon_0 \frac{I(t)}{\varepsilon_0 A} \underbrace{\int_{S_2} dS}_{A} = \mu_0 I$$

As expected.