Resistance

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In most material the induced emf is related to the current that flows in the circuit by Ohm's law, which is also valid in electrostatics

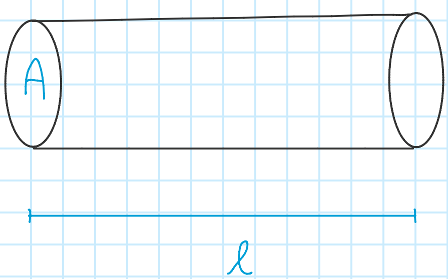
$$\mathcal{E} = RI$$

Where R is the resistance of the circuit. Notice however that, while

$$\mathcal{E} = \int \bar{E} \cdot d\bar{\ell}$$

If the induced emf is due to a changing magnetic field, E is not a conservative field and we can't say that E is minus the gradient of the scalar potential.

The resistance of a straight conductor of constant section is



$$R = \rho \frac{L}{A} \qquad [\rho] = \Omega\, m$$

ρ = RESISTIVITY (property of a given material)

One can also define the conductivity as the inverse of the resistivity

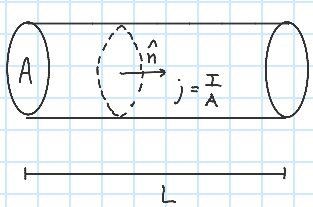
$$\sigma = \frac{1}{\rho} \qquad \sigma = \text{CONDUCTIVITY}$$

Ohm's law can be written in general terms as $\bar{J} = \sigma \bar{E}$

We can quickly check that for the case of a straight conductor of constant section one can recover the expression that we had above:

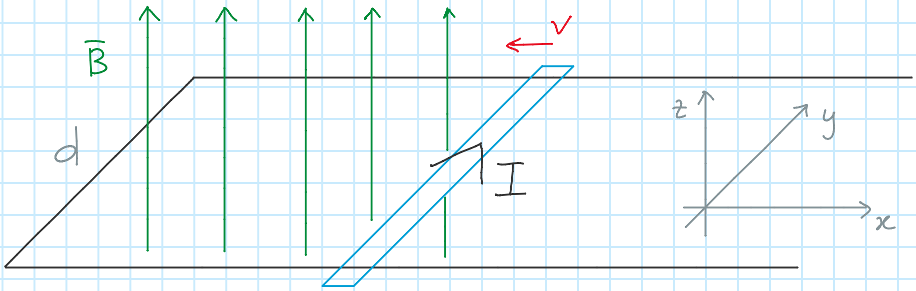
$$\mathcal{E} = \int \bar{E} \cdot d\bar{\ell} = \frac{1}{\sigma} \int \bar{J} \cdot d\bar{\ell} = \frac{1}{\sigma} \int \frac{I}{A} \hat{n} \cdot d\bar{\ell} = \frac{1}{\sigma A} I \int d\ell = \frac{IL}{\sigma A}$$

$$= \underbrace{\frac{\rho L}{A}}_{R} I = RI$$



Example: Loop of changing area in a constant magnetic field

Here we analyze again the example of a rectangular circuit with a that moves in a constant magnetic field perpendicular to the circuit.



The force acting on the blue moving bar is

$$\bar{F} = I\,\bar{d} \times \bar{B} = I\,d\,B\,\hat{j} \times \hat{k} = I\,d\,B\,\hat{i}$$

From Newton's second law we have the position of the bar along the x axis must satisfy the relation

$$m\ddot{x} = I\,B\,d$$

If there is no battery, the current I is due to the induced emf. Consequently the current will depend on the velocity of the bar along x.

$$\mathcal{E} = -\frac{d\phi}{dt} = -B\,d\,v = -B\,d\,\dot{x}$$

If we goes opposite to the x axis, the emf is positive, i.e. Tends to increase the current I= emf/R

$$I = \frac{\mathcal{E}}{R} = -\frac{B\,d}{R}\dot{x}$$

By plugging this expression for the current in Newton's second law one finds.

$$m\,\ddot{x} = -\frac{B^2 d^2}{R}\dot{x} \longrightarrow \frac{dv}{dt} = -\frac{B^2 d^2}{mR} v$$

$$\frac{dv}{v} = -\frac{B^2 d^2}{mR} dt \longrightarrow \ln v - \ln v_0 = -\frac{B^2 d^2}{mR} t$$

$$\boxed{v = v_0 \, e^{-\frac{B^2 d^2}{mR} t}}$$

No matter if the bar is moving initially to the left (v_0 < 0) or to the right (v_0 > 0) the bar will slow down and stop.

One can consider also the case in which there is a battery that provides an additional voltage V connected to the circuit. In that case

$$\mathcal{E} = V + \mathcal{E}_{induced} = V - Bd\dot{x}$$

$$I = \frac{V}{R} - \frac{Bd}{R}\dot{x}$$

$$m\ddot{x} = \frac{Bd}{R}\left(V - Bd\dot{x}\right) \longrightarrow \frac{dv}{dt} = \frac{Bd}{mR}\left(V - Bdv\right)$$

$$v = \frac{V}{Bd} + e^{-\frac{B^2d^2}{mR}t} C_1$$

In order to fix the integration constant C_1 one must impose that at t=0 the velocity is v_0.

$$v_0 = \frac{V}{Bd} + C_1 \longrightarrow C_1 = v_0 - \frac{V}{Bd}$$

$$v = \frac{V}{Bd} + e^{-\frac{B^2d^2}{mR}t}\left(v_0 - \frac{V}{Bd}\right)$$

$$\boxed{v = v_0 e^{-\frac{B^2d^2}{mR}t} + \frac{V}{Bd}\left(1 - e^{-\frac{B^2d^2}{mR}t}\right)}$$

As t grows the exponentials become smaller and smaller. The velocity will reach a constant value V/(Bd).