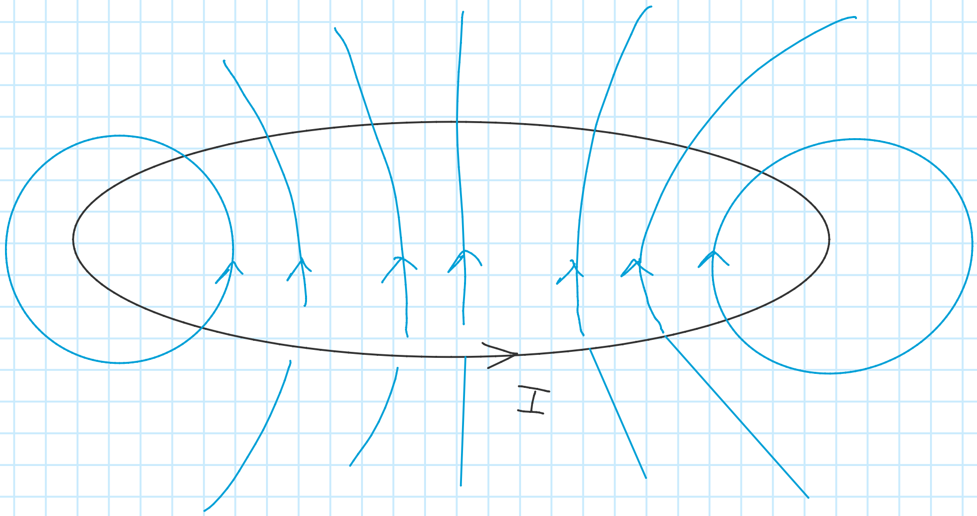
Inductance

Saturday, May 4, 2019 4:02 PM

Suppose that we consider a conducting loop; indicated by C, carrying a current I. The current creates a flux through the surface bound by C. If the current grows, the flux also grows. In turn, the change in flux creates an induced current through C, that sums to the primary current. The induced current will of course create its own contribution to the magnetic field, that will try to oppose the change in flux. The induced current will resist the increase of the primary current.

In order to build up the current I it will be necessary to overcome the induced emf. This costs energy.



$$\text{if} \quad \frac{dI}{dt} \neq 0 \rightarrow \frac{dB}{dt} \neq 0 \rightarrow \frac{d\phi_B}{dt} \neq 0 \rightarrow \mathcal{E} = -\frac{d\phi_B}{dt} \neq 0$$

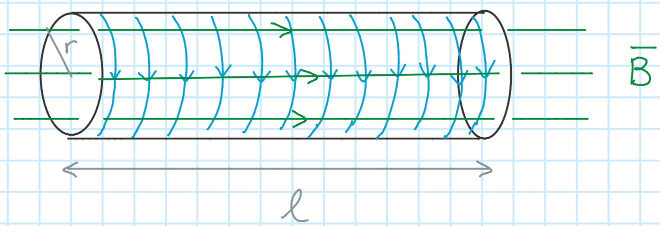
$\boxed{\text{Inductance}}$

The inductance, L, of the loop is defined as the ratio between the magnetic field flux going through the loop and the current in the loop. L depends only on the geometry of the circuit.

$$\phi_B = \int_{\partial S} \bar{B} \cdot d\bar{s} \qquad\qquad L \equiv \frac{\phi_B}{I}$$

It is instructive to consider as an example the inductance of a long solenoid

$r << \ell$



inside the solenoid

$$B = \mu_0 N I$$

N = # of turns per unith length

He flux of the magnetic field through a single loop in the solenoid is

$$\phi_1 = \pi r^2 \mu_0 N I$$

Since the total number of turns in a solenoid is Nl, the total magnetic field flux through the solenoid is

$$\phi_B = N \ell \phi_1 = \pi r^2 \mu_0 N^2 I \ell = \mu_0 N^2 I \underbrace{\pi r^2 \ell}_{V \,=\, \text{volume}} = \mu_0 N^2 I V$$

Therefore

$$L = \frac{\phi_B}{I} = \mu_0 N^2 V$$

depend only on the solenoid's geometry