Faraday's law of induction

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We saw that for fields that do not vary in time Maxwell's equations for the electric and magnetic fields decouple. A hint to the connection between the two can still be found in the fact that the sources of both E and B are charges, stationary charges are the sources of electrostatic fields, while steady currents are the sources of the magnetic fields.

For fields that vary in time, the connection between electric and magnetic fields is much deeper and more evident. Faraday's law states that

$$\nabla \times \bar{E} + \frac{\partial \bar{B}}{\partial t} = 0$$

Faraday's law says that a change in the magnetic field creates an electric field. The electric field can be employed to accelerate charges. In other words, the induced electric field creates a current. A current ultimately due to a varying magnetic field is called an induced current.

In the context of the general introduction to Maxwell's equation we already discussed how to rewrite this equation in integral form. We quickly repeat the argument here. Consider a conducting wire whose shape does not change in time. The wire describes a path that we label with the letter C. Let's then integrate the curl of E over a surface delimited by the path C.

$$\int_S (\nabla \times \bar{E}) \cdot d\bar{s} = \int_S -\frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{s} \equiv -\frac{d}{dt} \Phi_{\bar{B}}$$

we are assuming that S does not depend on time

In addition one can simply apply Stoke's theorem and find

$$\int_S (\nabla \times \bar{E}) \cdot d\bar{s} = \oint_C \bar{E} \cdot d\bar{\ell} \equiv \mathcal{E}$$

ELECTROMOTIVE FORCE

oriented according to the r.h.r.

$$\boxed{\mathcal{E} = -\frac{d}{dt}\phi_B}$$

FARADAY'S LAW (integral form)

Observe that for a conductor C that changes shape in time the definition of the electromotive force will need to be modified and becomes

$$\oint_{c(t)} d\bar{\ell} \cdot \left(\bar{E} + \bar{v}_e \times \bar{B}\right) \equiv \mathcal{E}$$

velocity of the electrons in the magnetic field

With the modified definition for the electromotive force, Faraday's law in integral form remains valid also for a conducting circuit that changes shape and/or moves in time.

We know from the introductory physics courses how to observe experimentally this effect: One can move a magnet in the vicinity of a conducting wire, one can change the current in a circuit in the vicinity of a conducting wire etc.

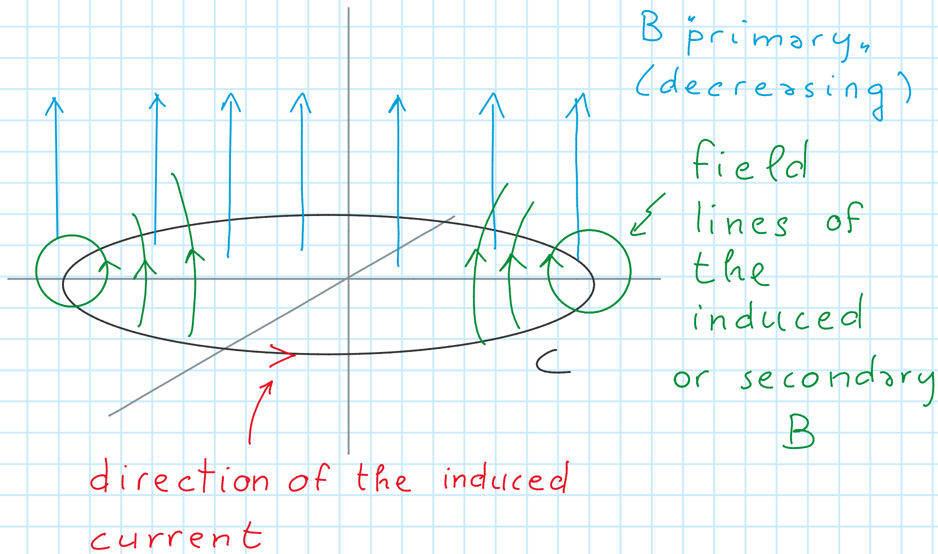
Lenz's law

The induced current creates its own magnetic field, this field is such that it opposes the change in magnetic flux that creates the induced current. Since the direction of the magnetic field due to the induced current depends on the polarity of the e.m.f., Lenz's law is encoded in the minus sign in Faraday's law.

$$\mathcal{E} = - \frac{d\phi_B}{dt}$$

LENZ'S LAW

More practically, one can visualize the direction of the induced current using the right hand rule. Consider for example a decreasing magnetic field perpendicular to the plane that contains a loop of conducting wire, the direction of the induced current will be such that the magnetic field due to the induced current tries to compensate for the decrease in flux of the "primary" magnetic field.



Changes in shape of the circuit

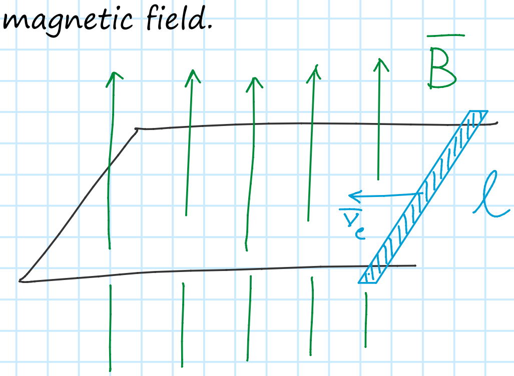
Now we want to consider the case in which the emf is due exclusively to the change in shape of the loop of conducting wire, i.e. only to the second term in

$$\mathcal{E} = \oint_{c(t)} d\bar{\ell} \cdot \left(\bar{E} + \bar{v}_e \times \bar{B} \right)$$

Remember that the E in the equation above arose from the fact that B was varying in time and that Faraday's law tells us that

$$\nabla \times \bar{E} + \frac{\partial \bar{B}}{\partial t} = 0$$

Let's than consider the classic case in which a conducting wire perpendicular to a constant magnetic field moves with constant velocity in a direction perpendicular to the wire and the magnetic field.



If B is constant there is no E due to the time variation of E and the equation for the emf becomes

$$\mathcal{E} = \oint_{C(t)} d\bar{\ell} \cdot (\bar{v}_e \times \bar{B}) = v \ell B$$

The emf above is indeed equal to minus the rate of change of the area in the loop over time, which is in turn related to the rate of change of the flux of B

$$\frac{d\phi_B}{dt} = B \frac{dA}{dt} = - B \frac{\ell v dt}{dt} = -B \ell v$$

the area decreases over time

$$\mathcal{E} = - \frac{d\phi_B}{dt} \quad \checkmark$$