Magnetic susceptibility and permeability

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For paramagnetic and diamagnetic materials the fields B and M are proportional to each other. However, while for paramagnetic materials the vectors B and H are parallel, for diamagnetic materials the vectors B and H are antiparallel. In those cases one can define the magnetic susceptibility such that

$$\boxed{\bar{M} = \chi_m \bar{H}}$$

$\chi_m \equiv$ magnetic susceptibility

The magnetic susceptibility can be either positive or negative.
One can then insert the above in the relation between H and B

$$\bar{B} = \mu_0 (\bar{H} + \bar{M}) = \underbrace{\mu_0 (1 + \chi_m)}_{\equiv \mu} \bar{H} \equiv \mu \bar{H}$$

$\boxed{\equiv \mu \text{ magnetic permeability}}$

In vacuum μ = μ_0; this explains why μ_0 is called the permeability of free space.
(there is no magnetization in vacuum.)

Observe that even in linear media the fact that the divergence of B is zero does not imply that the divergence of H is zero:

$$\underbrace{\bar{B} = \mu \bar{H}}_{\text{linear medium}} \qquad \nabla \cdot \bar{B} = 0 \quad \not\Rightarrow \quad \nabla \cdot \bar{H} = 0$$

$$\nabla \cdot \bar{H} = \nabla \cdot \left(\frac{1}{\mu} \bar{B} \right) = \frac{1}{\mu} \nabla \cdot \bar{B} + \bar{B} \cdot \left(\nabla \frac{1}{\mu} \right) = \bar{B} \cdot \left(\nabla \frac{1}{\mu} \right)$$

H is not divergenceless at points where μ is changing (for example at a boundary between two media)

Observe that within a linear material

$$\bar{J}_b = \nabla \times \bar{M} = \nabla \times (\chi_m \bar{H}) = \chi_m \nabla \times \bar{H} = \chi_m \bar{J}_f$$

$$\bar{J}_f = 0 \implies \bar{J}_b = 0$$

All of the bound current will be on the surface.
A very similar thing happens in electrostatics

$$\rho_b = -\nabla \cdot \bar{P} = -\nabla \cdot (\varepsilon_0 \chi_e \bar{E}) = -\nabla \cdot \left(\frac{\varepsilon_0}{\varepsilon} \chi_e \bar{D}\right)$$

$$= -\frac{\varepsilon_0 \chi_e}{\varepsilon} \underbrace{\nabla \cdot \bar{D}}_{\rho_f} = -\frac{\varepsilon_0}{\varepsilon} \chi_e \rho_f$$

Remember that

$$\varepsilon = \varepsilon_0 (1 + \chi_e) \longrightarrow \chi_e = \frac{\varepsilon}{\varepsilon_0} - 1$$

$$\frac{\varepsilon_0}{\varepsilon} \chi_e = 1 - \frac{\varepsilon_0}{\varepsilon}$$

$$\boxed{\rho_b = -\left(1 - \frac{\varepsilon_0}{\varepsilon}\right) \rho_f}$$

Therefore, if

$$\rho_f = 0 \implies \rho_b = 0$$

In the bulk of a uniform linear dielectric, the bound charge shadows the free charge so that

$$\rho_{TOT} = \rho_b + \rho_f = \left(-1 + \frac{\varepsilon_0}{\varepsilon} + 1\right) \rho_f = \frac{\varepsilon_0}{\varepsilon} \rho_f$$

If the free charge is simply a point charge placed in the dielectric, the field in the dielectric differs from the field in vacuum because of the permittivity

$$E_{vac} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \implies E_{diel.} = \frac{1}{4\pi\varepsilon} \frac{Q}{r^2}$$