Equipotential surfaces - field lines

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An equipotential surface is a two dimensional surface in three-dimensional space defined by the equation

$$\varphi(\bar{r}) = \text{constant} \quad \forall\, \bar{r} \in S$$

By definition then, the work done by the field when a test charge moves from one point on an equipotential surface to another point on the same surface is zero.

Furthermore, since

$$\varphi(\bar{r}_B) - \varphi(\bar{r}_A) = -\int_{\bar{r}_A}^{\bar{r}_B} d\bar{\ell} \cdot \bar{E}$$

If both A and B are on the same equipotential surface one has

$$\int_{\bar{r}_A}^{\bar{r}_B} d\bar{\ell} \cdot \bar{E} = 0$$

For every path between A and B and in particular for every path along the equipotential surface. Since we can take any two points on the surface, either E is zero everywhere on the surface or it is perpendicular to the surface everywhere.

Field lines are defined as lines which are everywhere tangent to the field, so that an infinitesimal length along the line can be parameterized as

$$d\bar{\ell} = dt\,\bar{E} \qquad t = \text{parameter}$$

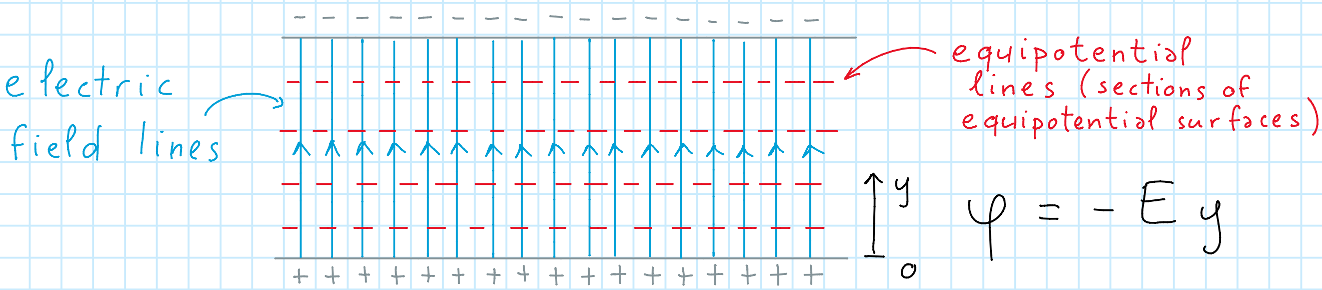
$$dt = \frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z}$$

Electric field lines cannot self intersect (the electric field cannot have two directions at the same point). Similarly two equipotential surfaces cannot intersect (a point in space cannot have two different potential values). Field lines are always perpendicular to equipotential surfaces. Field lines (like the field E) go from positive charges to negative charges.

Maxwell established a tradition of drawing a number of field lines per unit area which is proportional to the strength of the field. However, keep in mind that there is a field line (and an equipotential surface) going through each point in space.

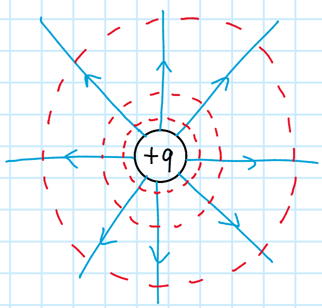
Classic examples are

<u>Parallel charged plates of uniform surface charge density</u>



$$\varphi = -Ey$$

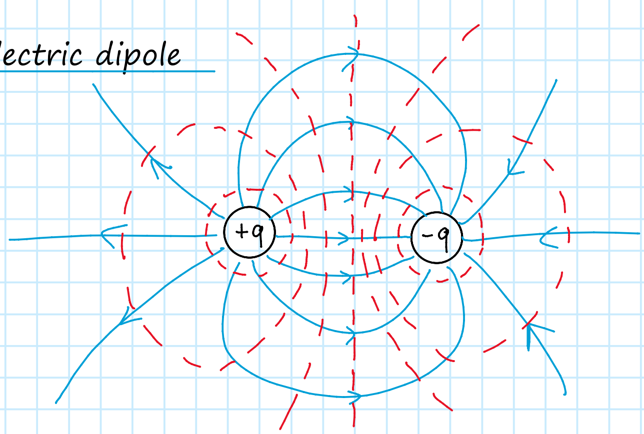
<u>Point charge</u>



$$\varphi = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r}$$

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}$$

<u>Electric dipole</u>



Example: Line segment with constant charge density