Polarization

Monday, February 11, 2019 3:38 PM

When a piece of material is placed in an electric field, the material typically behaves either as dielectric or as a conductor (See notes on conductors discussed earlier in the course.)

Dielectric (also called insulators): All electrons are tightly bound to individual atoms and molecules

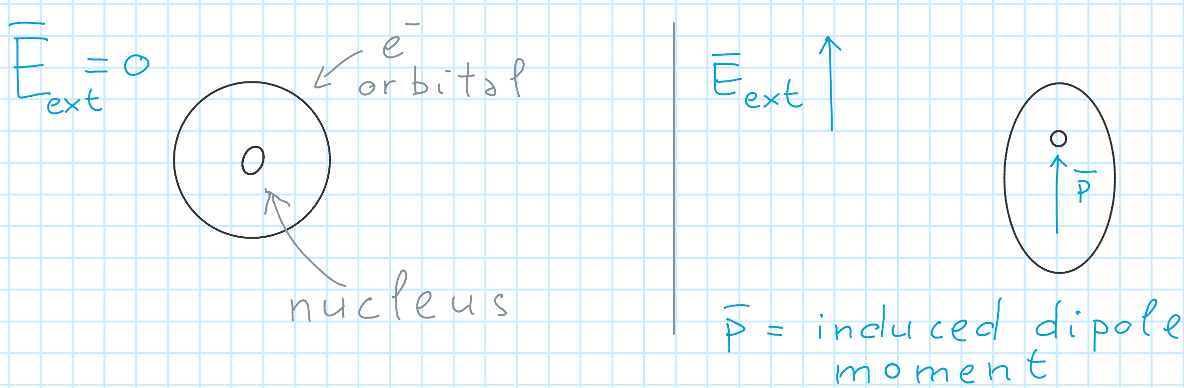
Conductors: Contain an "unlimited" amount of free charges which are free to move about through the material (ex. Electrons in the conduction band in metals)

There are two mechanisms through which an electric field can alter the charge distribution in a dielectric atom or molecule: By "stretching" the molecule or by rotating it.

Induced dipoles

An external electric field applies on electrons and nuclei forces which pull in opposite directions, since electrons have negative charge and nuclei have positive charge. If the field is very strong, electrons can be completely separated from nuclei (ionization). If the field is not sufficiently strong to ionize an atom or molecule, it induces a dipole moment in the atom/molecule.

Effect of E on a neutral atom (schematically)



$$\boxed{\bar{p} = \alpha \bar{E}}$$

α = atomic polarizability

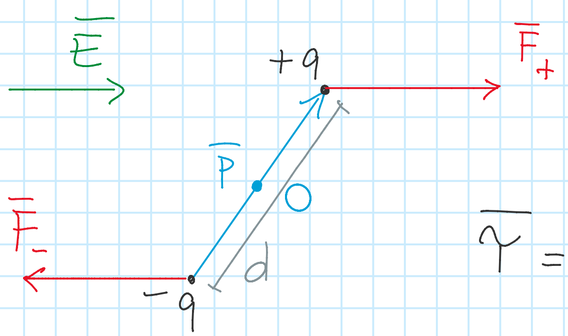
For molecules things are more complicated and in general

$$\bar{p} = \alpha \cdot \bar{E}$$

Where alpha is a 3X3 matrix, the polarizability tensor.

Alignment of polar molecules

Some molecules have permanent dipole moments (ex water, p ~ 6.1 x 10^(-30) C m, electrons tend to cluster around the oxygen atom). These molecules are called polar molecules. The net force of an external electric field on the dipole is zero (if the field is uniform over the dipole length). However, the external electric field applies a torque on the dipole. This torque tries to align the dipole to the field.



$$p = qd$$

$$\bar{\tau} = \frac{\bar{d}}{2} \times q\bar{E} + \left(-\frac{\bar{d}}{2}\right) \times (-q\bar{E})$$

$$= q\bar{d} \times \bar{E} = \bar{p} \times \bar{E}$$

if $\bar{p} \,//\, \bar{E} \;\rightarrow\; \bar{\tau} = 0$

Polarization

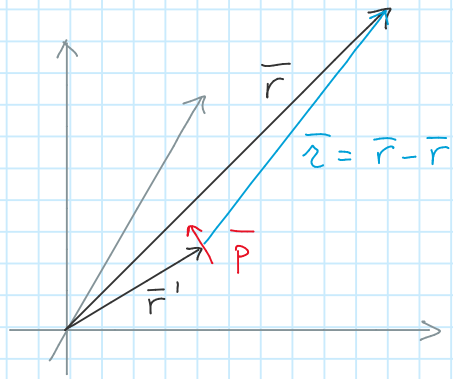
The two phenomena described above amount to the same thing: Microscopic dipoles align with the external field, i.e. the material becomes polarized. One can then define the polarization vector

$\bar{P} =$ dipole moment per unit of volume

$$[P] = \frac{C\,\cancel{m}}{m^{\cancel{3}\,2}} = \frac{C}{m^2}$$

Therefore there will a contribution to the electric field due to the fact that the material is polarized. This field needs to be added to the external field that induced the polarization in the dielectric material. The field produced by an individual dipole is

$$\varphi(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{(\bar{r}-\bar{r}')\cdot\bar{p}}{|\bar{r}-\bar{r}'|^3} = \frac{1}{4\pi\varepsilon_0}\,\frac{\hat{\mathscr{r}}\cdot\bar{p}}{\mathscr{r}^2} \qquad \bar{\mathscr{r}} \equiv \bar{r}-\bar{r}'$$



In a material, the infinitesimal amount of dipole moment in an infinitesimal volume is

$$d\bar{p} = d^3r'\ \bar{P}(\bar{r}')$$

$$\varphi(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\int_V d^3r'\ \frac{\bar{P}(\bar{r}')\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}$$

Now let's observe that

$$\nabla_{\bar{r}'}\frac{1}{\mathscr{r}} = \nabla_{\bar{r}'}\left(\frac{1}{|\bar{r}-\bar{r}'|}\right) = -\frac{1}{\mathscr{r}^2}\nabla_{\bar{r}'}|\bar{r}-\bar{r}'|$$

$$= -\frac{1}{2\mathscr{r}^3}\ 2(x_i - x_i')(-1)\,\hat{\imath}_i = +\frac{\bar{\mathscr{r}}}{\mathscr{r}^3} = +\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

Therefore

$$\varphi(\bar r) = \frac{1}{4\pi\varepsilon_0}\int_V \bar P(\bar r') \cdot \nabla_{\bar r'}\left(\frac{1}{\tau}\right) d^3r'$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_V \nabla' \cdot \left(\frac{\bar P}{\tau}\right) d^3r' - \int_V \frac{1}{\tau}\left(\nabla_{\bar r'} \cdot \bar P\right) d^3r'\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\oint_{\partial V} \frac{1}{\tau}\underbrace{\bar P \cdot d\bar s}_{\substack{\text{dimensionally}\\ \text{identical to } \sigma ds}} - \int_V \frac{1}{\tau}\left(\underbrace{\nabla_{\bar r'} \cdot \bar P}_{\substack{\text{dimensionally}\\ \text{identical to } \rho}}\right) d^3r'\right]$$

Let's then introduce the bound charge and surface density

$$\boxed{\sigma_B \equiv \bar P \cdot \hat n} \qquad \boxed{\rho_B \equiv -\nabla \cdot \bar P}$$

($\hat n$ is a unit vector in the direction of $d\bar s$)

One can then finally write

$$\boxed{\varphi(\bar r) = \frac{1}{4\pi\varepsilon_0}\left[\oint_{\partial V} \frac{\sigma_b}{\tau}\, ds + \int_V \frac{\rho_b}{\tau}\, d^3r'\right]}$$