

Differential equations satisfied by Jacobi elliptic functions

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By using Mathematica (file Jacobi Elliptic functions) I determined what are the differential equations satisfied by the 12 Jacobi elliptic functions. The general form of the equation is

$$\left(\frac{dz_n}{du} \right)^2 = \alpha z_n^4(u, k) + \beta z_n^2(u, k) + \gamma$$

Where z_n represents one of the 12 Jacobi functions. In the following I employ the mathematica notation so that the second argument of the function is taken to be $m = k^2$

$$z_n \in \{ sn, cn, dn, ns, nc, nd, sc, sd, cd, cs, ds, dc \}$$

The constants for the various choices of z_n are

	α	β	γ
sn	m	$-(m+1)$	1
cn	$-m$	$(2m-1)$	$1-m$
dn	-1	$(2-m)$	$m-1$
ns	1	$-(m+1)$	m
nc	$1-m$	$(2m-1)$	$-m$
nd	$m-1$	$(2-m)$	-1
sc	$1-m$	$(2-m)$	1
sd	$m(m-1)$	$(2m-1)$	1
cd	m	$-(m+1)$	1
cs	1	$(2-m)$	$1-m$
ds	1	$(2m-1)$	$m(m-1)$
dc	1	$-(1+m)$	m