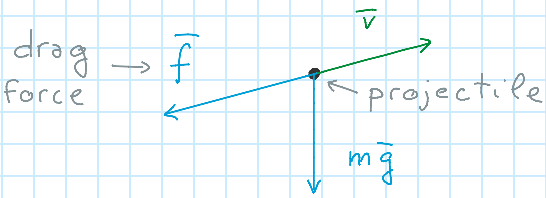
Air resistance

Sunday, June 16, 2019 4:06 AM

The purpose of this section is to study the motion of a projectile including air resistance, which is typically neglected in introductory mechanics courses. Air resistance (or the resistance of another gas or fluid through which the projectile moves) is called drag force. It follows from experience that the drag force grows with the velocity of the projectile. Most of the times, the drag force acts in the direction opposite to the velocity v. (There are exceptions to this, for example the lift force acting on the wings of an airplane.)

In these notes we assume that the drag force points opposite to the velocity of the projectile.



$$\bar{f} = -f(v)\,\hat{v} \qquad \hat{v} = \frac{\bar{v}}{|\bar{v}|}$$

The scalar function f(v) is a complicated function of the velocity, in particular when v is close to the speed of sound in the fluid. However, at smaller velocities the function f is well modeled by the sum of a linear and a quadratic term:

$$f(v) = \underbrace{bv}_{f_{lin}} + \underbrace{cv^2}_{f_{quad}}$$

f_{lin} → proportional to viscosity ("thickness") of the fluid and to the linear dimensions of the projectile

f_{quad} → proportional to the density of the medium and to the square of the linear dimensions of the projectile

For a spherical projectile one has

$$b = \beta D \qquad c = \gamma D^2$$

D = projectile's diameter

In air at STP (1 atm, 20 Celsius)

$$\beta = 1.6 \times 10^{-4} \frac{N\,s}{m^2} \qquad \gamma = 0.25 \frac{N\,s^2}{m^4}$$

If, as it is often the case, one can neglect one of the two terms in f it becomes easier to solve the differential equations derived from Newton's second law. In order to determine when this is possible, one needs to consider the ratio between the linear and the quadratic term.

$$\frac{f_{quad}}{f_{lin}} = \frac{c\,v^2}{b\,v} \overset{\text{spherical projectile}}{=} \frac{\gamma}{\beta} D v \underset{\text{in air}}{=} \left(1.6 \times 10^{3} \frac{s}{m^2}\right) D v$$

Example. A baseball and a rain drop

Compare the impact of the linear and quadratic terms in the air drag for a baseball and a drop of rain.

baseball $\quad D = 7\ cm \quad v = 5\ m/s$

$$\frac{f_{quad}}{f_{lin}} = \left(1.6 \times 10^{+3} \frac{s}{m^2}\right) 7 \times 10^{-2}\, m \; 5 \frac{m}{s} \simeq 600$$

For baseballs and objects of similar size and speed one can safely neglect the linear part in the air drag formula and simply use

$$\bar{f} = -c v^2 \hat{v} \underset{\text{spherical projectile}}{=} -\gamma D^2 v^2 \hat{v}$$

Rain drop

$D \simeq 1\,mm \quad v \simeq 0.6 \frac{m}{s}$

$$\frac{f_{quad}}{f_{lin}} = \left(1.6 \times 10^{+3} \frac{s}{m^2}\right) 10^{-3}\, m \; 0.6 \frac{m}{s} \simeq 1$$

Therefore for a rain drop both contributions to the air drag are equally important and must be accounted for. This makes the solution of Newton's second law more complicated.

For drops which are much smaller and slower than a rain drop, such as an oil drop in the Millikan experiment (experiment used to determine then electron charge, where an electrostatic force cancels the weight of the charged oil drops) the ratio is

$$\frac{f_{quad}}{f_{lin}} \simeq 10^{-7} \longrightarrow \bar{f} = -bv\hat{v} = -b\bar{v}$$

We already started that beta is proportional to the fluid viscosity, while gamma is proportional to the fluid density, therefore

$$\frac{f_{quad}}{f_{lin}} = \frac{\gamma}{\beta} D v \propto \frac{\rho}{\eta} D v \equiv R$$

Reynolds number

viscosity

R is dimensionless.

$R \gg 1 \longrightarrow f_{quad}$ dominates

$R \ll 1 \longrightarrow f_{lin}$ dominates