Harmonic oscillator phase space

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The simplest example of phase space is probably the phase space of a harmonic oscillator. The Hamiltonian of a mass m moving in one dimension and subject to Hooke's law is

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2} k x^2 = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 \qquad \omega = \sqrt{\frac{k}{m}}$$

Hamilton's equations are

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial p} = \frac{p}{m} \qquad \dot{p} = -\frac{\partial \mathcal{H}}{\partial x} = -m \omega^2 x$$

These equations are equivalent to

$$\ddot{x} = -\omega^2 x$$

The solution is the usual

$$x = A \cos(\omega t - \delta) \longrightarrow p = m\dot{x} = -m\omega A \sin(\omega t - \delta)$$

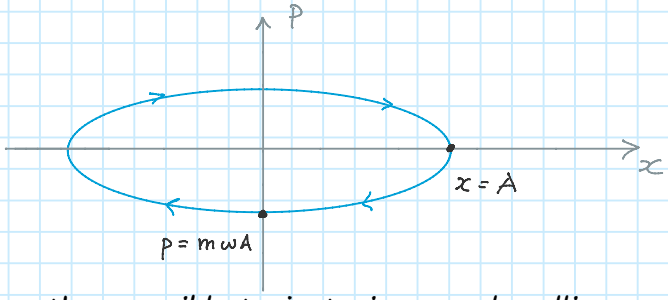
In addition, the Hamiltonian is the total energy of the system

$$\frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 = \frac{1}{2} m \omega^2 A^2$$

A is the max amplitude of the oscillation

$$\frac{p^2}{m^2 \omega^2 A^2} + \frac{x^2}{A^2} = 1$$

THE PHASE SPACE TRAJECTORIES ARE ELLIPSES



All of the other possible trajectories are also ellipses, which are either inside or outside the one shown in then figure, different possible trajectories do not intersect each other.