2nd law in cartesian coordinates

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The solution of every problem in classical mechanics starts by choosing a frame of reference and a set of coordinates to describe the motion of an object in that frame.

Newton's second law is a second order vector differential equation. By projecting out the various coordinates one can find a set of second order differential equations for scalar quantities.

The first system of coordinates that is analyzed in every course and book is the cartesian system of coordinates. For cartesian coordinates newton's second law becomes

$$m\ddot{\bar{r}} = \bar{F}$$

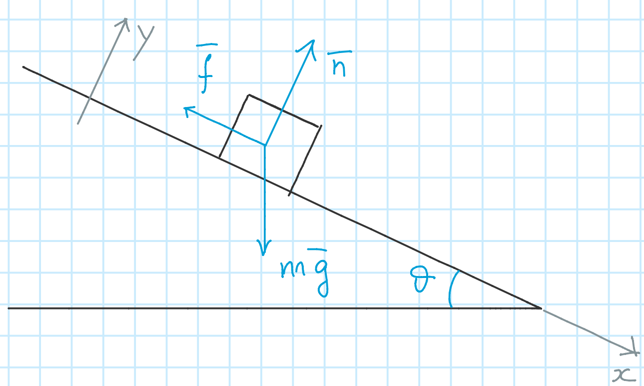
$$\bar{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} \qquad \bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\boxed{\begin{aligned} m\ddot{x} &= F_x \\ m\ddot{y} &= F_y \\ m\ddot{z} &= F_z \end{aligned}}$$

Newton's 2nd Law in cartesian coordinates

Example: Block sliding down an inclined plane

A classic problem in Newtonian mechanics is to describe the motion of a block along an inclined plane. It is of course useful to choose an axis along the inclined plane



No acceleration along y

$$m\ddot{y} = 0 = n - mg\cos\vartheta$$

Along x

$$m\ddot{x} = mg\sin\vartheta - f_K = mg\sin\vartheta - \mu_K \overbrace{mg\cos\vartheta}^{n}$$

Differential equation to solve

$$\boxed{\ddot{x} = g(\sin\vartheta - \mu_K\cos\vartheta)}$$

equation of motion

$$\dot{x} = g(\sin\vartheta - \mu_K\cos\vartheta)\,t$$

initial condition

$\dot{x}(0) = 0$

$$\boxed{x(t) = \frac{1}{2}g(\sin\vartheta - \mu_K\cos\vartheta)\,t^2}$$

SOLUTION OF THE EQ. OF MOTION

initial condition $x(0) = 0$