Euler Lagrange equations in configuration space

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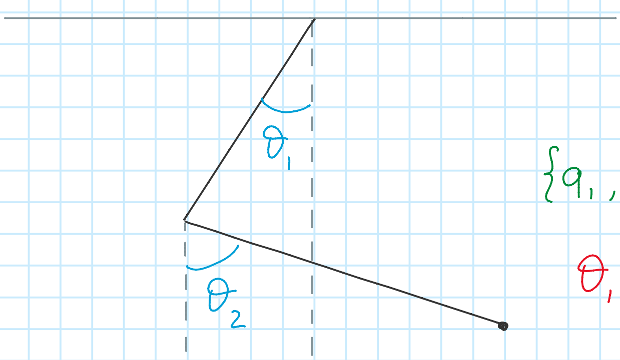
The language of calculus of variations can be applied to the study of mechanics. This is precisely what is done in Lagrangian Mechanics. Before looking at which kind of physical variational principle describes mechanics, it is useful to adapt the notation developed so far to the needs of the study of mechanics.

The role of the integration parameter will be played by time, t. The functions are the coordinates that describe the status of the system, called "configuration". The number and nature of these coordinates, indicated in general by the letter q with a numerical subscript, depends on the system. For example, for a single free particle described in cartesian coordinates, the configuration of the system at any given moment is specified by the three cartesian coordinate x,y,z. However, one can decide to describe the configuration of the system in spherical coordinates, so that the coordinates will be r, theta and phi. Notice that in that case not all coordinates have the same dimensions.

There are cases in which the system is subject to some constraints so that one does not need to consider three coordinates for each point of interest in the system. For example, in a double pendulum (a simple pendulum attached to the end of another simple pendulum) one only needs two angles to specify the configuration of the system. For these reasons the qs are called generalized coordinates. The space spanned by the generalized coordinates is called configuration space.



DOUBLE PENDULUM

$$\{q_1, q_2\} \longrightarrow \{\vartheta_1, \vartheta_2\}$$ GENERALIZED COORDINATES

$$\vartheta_1, \vartheta_2 \in [0, 2\pi]$$ CONFIGURATION SPACE

The functional of the generalized coordinates that is needed in mechanics will be called the Lagrangian of the system

$$\mathcal{L} = \mathcal{L}(\underbrace{q_1, \dots, q_n}_{\text{generalized coordinates}}, \underbrace{\dot{q}_1, \dots, \dot{q}_n}_{\text{generalized velocities}}, t)$$

$$\dot{q} = \frac{dq}{dt}$$

The integral to which the variational principle is applied is called the action S

$$S = \int_{t_1}^{t_2} \mathcal{L}(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n, t)\, dt$$

In this case one will have n Euler Lagrange equations (usually called simply Lagrange equations in mechanics)

$$\boxed{\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = 0}$$