Atwood's machine

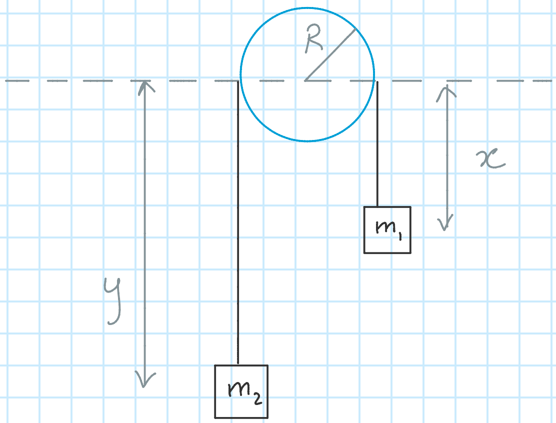
Monday, September 2, 2019 9:49 AM

Let's consider an Atwood's machine as an example of a constrained system.

length of rope

$$L = x + y + 2\pi R$$

$$x = -y + \text{const}$$



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{y}^2 = \frac{1}{2}(m_1 + m_2)\dot{x}^2$$

$$U = -m_1 g x - m_2 g y = (m_2 - m_1) g x + \underbrace{\text{const}}_{\text{set} = 0}$$

$$\mathcal{L} = \frac{1}{2}(m_1 + m_2)\dot{x}^2 - (m_2 - m_1) g x$$

Lagrange's equation of motion is then

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = -(m_2 - m_1) g - (m_1 + m_2)\ddot{x} = 0$$

$$\ddot{x} = -\underbrace{\frac{m_2 - m_1}{m_1 + m_2} g}_{\text{constant acceleration}}$$

SAME RESULT OBTAINED WITH NEWTON'S LAWS

It should be observed that no reference was made to the force of constraint, namely the tension in the rope.
In this case it was easy to solve the problem with Newtonian methods. The Lagrangian approach leads quickly to the equations of motion even in cases in which the Newtonian approach becomes very cumbersome.