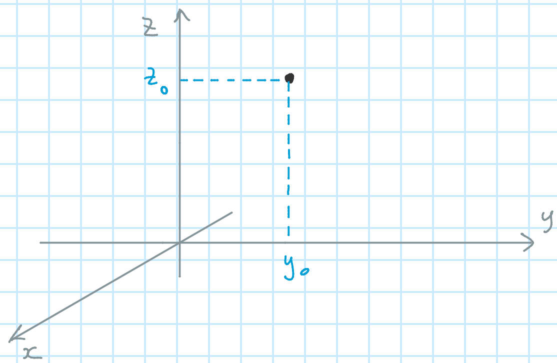
Simple moments and products of inertia

Tuesday, November 19, 2019 10:02 AM

Example 1

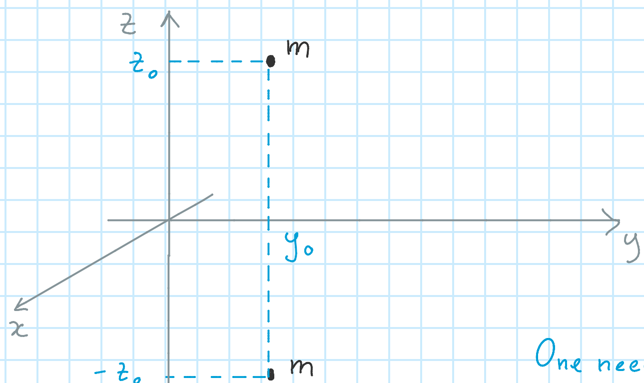
Calculate the moment and products of inertia about the z axis of a single mass m located on the y z plane



$$I_{xz} = m \cancel{x_o}^{\,0} z_o = 0 \qquad I_{yz} = m y_o z_o \qquad I_{zz} = m y_o^2$$

Example 2

Same as above but with a second identical mass placed symmetrically below the x y plane.



$$I_{xz} = m \cancel{x_o}^{\,0} z_o - m \cancel{x_o}^{\,0} z_o = 0$$

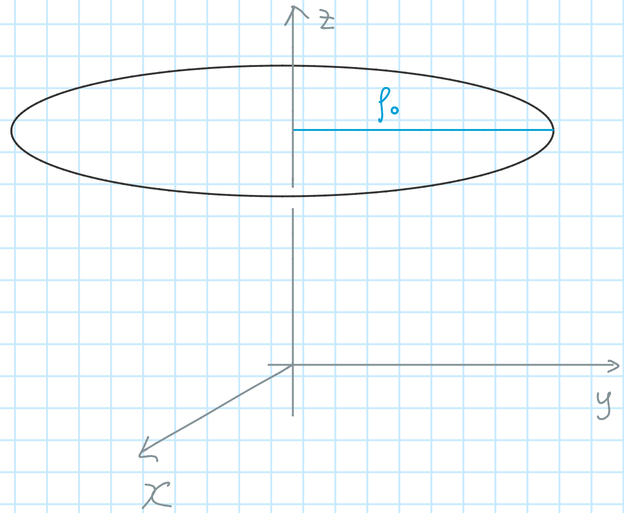
$$I_{yz} = m y_o z_o - m y_o z_o = 0$$

$$I_{zz} = m y_o^2 + m y_o^2 = 2 m y_o^2$$

One needs to consider the contributions of two masses in the summations defining I_{ij}

Example 3

A uniform ring centered on the z axis and parallel to the x y plane.



Since in this case the mass distribution is continuous, the summations in the definitions of the product of inertia should be replaced by integrals

$$I_{xz} = \int x z \, dm = \int_0^{2\pi} \rho_0 \cos\phi \, z \, \frac{M}{2\pi \rho_0} \rho_0 \, d\phi = 0$$

$$I_{yz} = \int y z \, dm = \int_0^{2\pi} \rho_0 \sin\phi \, z \, \frac{M}{2\pi \rho_0} \rho_0 \, d\phi = 0$$

$$I_{zz} = \int (x^2 + y^2) \, dm = \int_0^{2\pi} \rho_0^2 \cos^2\phi \, \frac{M}{2\pi \cancel{\rho_0}} \cancel{\rho_0} \, d\phi + \int_0^{2\pi} \rho_0^2 \sin^2\phi \, \frac{M}{2\pi \rho_0} \rho_0 \, d\phi$$

$$= 2 \, \frac{M \rho_0^2}{2\pi} \underbrace{2\pi \, \frac{1}{2}}_{= \int_0^{2\pi} \sin^2\phi \, d\phi = \int_0^{2\pi} \cos^2\phi \, d\phi} = M \rho_0^2$$

This last example illustrates a general rule, if an object is symmetric with respect to its axis of rotation, the products of inertia about the axis of rotation are zero.