Particle on the surface of a cylinder

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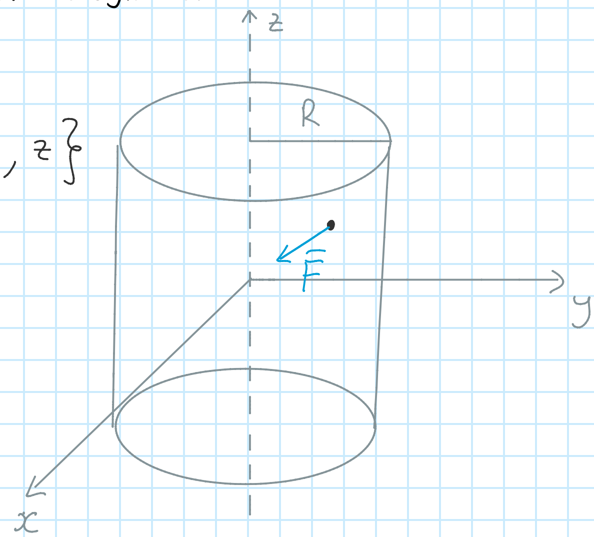
Consider a point particle moving on the surface of a cylinder and subject to an elastic force pointing toward the center of the cylinder

cylindrical coordinates $\{\rho, \phi, z\}$

$\rho = R$ always

$\bar{F} = -k\bar{r}$



$$T = \frac{1}{2} m \left((R\dot{\phi})^2 + \dot{z}^2 \right) \qquad U = \frac{1}{2} k r^2 = \frac{1}{2} k \left(R^2 + z^2 \right)$$

$$\mathcal{L} = \frac{1}{2} m \left(R^2 \dot{\phi}^2 + \dot{z}^2 \right) - \frac{1}{2} k \left(R^2 + z^2 \right)$$

TWO DEG OF FREEDOM

One needs to consider two Lagrange's equations

$$\frac{\partial \mathcal{L}}{\partial z} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{z}} = -kz - m\ddot{z} = 0 \longrightarrow \underbrace{m\ddot{z} = -kz}_{\text{harmonic motion}}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = -\underbrace{\frac{d}{dt} m R^2 \dot{\phi}}_{\text{conservation of angular momentum}} = 0$$

The particle moves with constant angular velocity and oscillates harmonically along z. The particle follows a sinusoidal trajectory on the cylinder's surface.