Work energy theorem

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The kinetic energy of a point particle is defined as

$$\boxed{T = \frac{1}{2} m v^2}$$

KINETIC ENERGY

m = particle's mass , v = particle's velocity

Let's consider the rate of change of the velocity in time

$$\frac{dT}{dt} = \frac{m}{2}\frac{d}{dt}(v^2) = \frac{m}{2}\frac{d}{dt}(\bar{v}\cdot\bar{v}) = \frac{m}{2}(\dot{\bar{v}}\cdot\bar{v} + \bar{v}\cdot\dot{\bar{v}})$$

$$= \underbrace{m\dot{\bar{v}}}_{\text{NEWTON'S 2nd LAW}}\cdot\bar{v} = \bar{F}\cdot\bar{v}$$

but $\bar{v}\,dt = d\bar{r}$

Consequently

$$\boxed{dT = \bar{F}\cdot d\bar{r}}$$

WORK ENERGY THEOREM (infinitesimal version)

$$\bar{F}\cdot d\bar{r} \equiv dW$$

infinitesimal amount of work done by the force $\bar{F}$

One can go from the infinitesimal version on the work energy theorem to a finite version by adding several infinitesimal contributions.

line integral

$$\Delta T \equiv T_2 - T_1 = \sum \bar{F}\cdot d\bar{r} \longrightarrow = \int_1^2 \bar{F}\cdot d\bar{r}$$

$$\boxed{T_2 - T_1 = \int_1^2 \bar{F} \cdot d\bar{r} = W(1 \to 2)}$$

WORK ENERGY THEOREM

(finite version)

The force in the integral in the work energy theorem is the net force acting on an object. Of course this force is in general the sum of many separate forces applied to the object

$$\bar{F} = \sum_{i=1}^{N} \bar{F}_i$$

$$W(1 \to 2) = \int_1^2 \bar{F} \cdot d\bar{r} = \int_1^2 \sum_i \bar{F}_i \cdot d\bar{r}$$

$$= \sum_i \int_1^2 \bar{F}_i \cdot d\bar{r} = \sum_i W_i(1 \to 2)$$

The total work is the sum of the work done by each of the forces acting on the object.