

# Work energy theorem

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The kinetic energy of a point particle is defined as

$$T = \frac{1}{2} m v^2$$

KINETIC  
ENERGY

$m$  = particle's mass,  $v$  = particle's velocity

Let's consider the rate of change of the velocity in time

$$\frac{dT}{dt} = \frac{m}{2} \frac{d}{dt} (v^2) = \frac{m}{2} \frac{d}{dt} (\vec{v} \cdot \vec{v}) = \frac{m}{2} (\dot{\vec{v}} \cdot \vec{v} + \vec{v} \cdot \dot{\vec{v}})$$

$$= m \dot{\vec{v}} \cdot \vec{v} = \vec{F} \cdot \vec{v}$$

NEWTON'S  
2nd LAW

$$\text{but } \vec{v} dt = d\vec{r}$$

Consequently

$$dT = \vec{F} \cdot d\vec{r}$$

WORK ENERGY  
THEOREM  
(infinitesimal  
version)

$$\vec{F} \cdot d\vec{r} \equiv dW$$

infinitesimal amount  
of work done by the force  $\vec{F}$

One can go from the infinitesimal version on the work energy theorem to a finite version by adding several infinitesimal contributions.

$$\Delta T \equiv T_2 - T_1 = \sum \vec{F} \cdot d\vec{r} \rightarrow = \int_1^2 \vec{F} \cdot d\vec{r}$$

line integral

$$T_2 - T_1 = \int_1^2 \vec{F} \cdot d\vec{r} = W(1 \rightarrow 2)$$

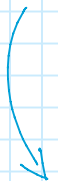
WORK ENERGY

THEOREM

(finite version)

The force in the integral in the work energy theorem is the net force acting on an object. Of course this force is in general the sum of many separate forces applied to the object

$$\vec{F} = \sum_{i=1}^N \vec{F}_i$$


$$\begin{aligned} W(1 \rightarrow 2) &= \int_1^2 \vec{F} \cdot d\vec{r} = \int_1^2 \sum_i \vec{F}_i \cdot d\vec{r} \\ &= \sum_i \int_1^2 \vec{F}_i \cdot d\vec{r} = \sum_i W_i(1 \rightarrow 2) \end{aligned}$$

The total work is the sum of the work done by each of the forces acting on the object.