Quadratic drag

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For baseball footballs and in general macroscopic objects at speeds that we encounter in everyday experience the drag force is quadratic in the speed

$$\bar{f} = -c v^2 \hat{v}$$

This makes life way more complicated than in the linear case, because the differential equations satisfied by the x and y component of the velocity are in this case non linear:

$$m\dot{\bar{v}} = m\bar{g} + \bar{f} \longrightarrow \begin{cases} m\dot{v}_x = -c\sqrt{v_x^2 + v_y^2}\, v_x \\ m\dot{v}_y = mg - c\sqrt{v_x^2 + v_y^2}\, v_y \end{cases}$$

(we assumed a y axis going downward)

In addition, the two equations are coupled. It is not possible to solve analytically this system of equations in terms of elementary functions. In general, non linear differential equations are more complicated to solve than linear equations. However, it is possible to solve the equations in the case in which the object is moving only horizontally or only vertically.

HORIZONTAL MOTION $\quad m\dot{v} = -cv^2$

VERTICAL MOTION $\quad m\dot{v} = mg - cv^2$