Space, Time, Mass, and Force

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Newtonian mechanics has the goal of describing the motion of object in space and time. It achieves this goal through Newton's three laws of motion. These laws involve the concepts of mass and force.

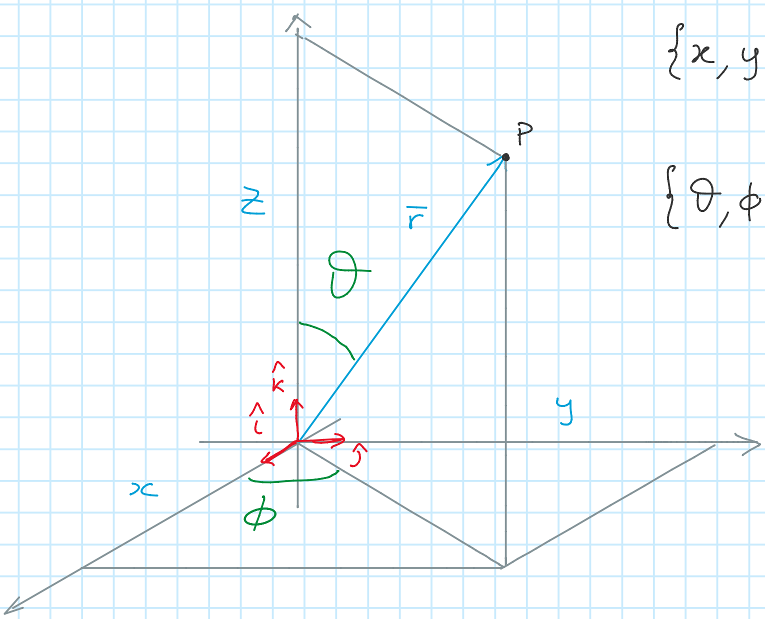
Space (review of vectors)

In order to describe the position of an object in space, it is useful to use the language of vectors, that we review very briefly here (see also notes used at the beginning of the Electricity and Magnetism course)

A point in space is indicated by

$$\bar{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$$

$$x = r \sin\vartheta \cos\phi$$
$$y = r \sin\vartheta \sin\phi$$
$$z = r \cos\vartheta$$



$\{x, y, z\}$ cartesian coordinates

$\{\vartheta, \phi, r\}$ polar coordinates

The unit vectors and the space coordinates are indicated in different ways in different books or situations, we need to be able to work with several of them

$\hat{\imath}$	$\hat{\jmath}$	$\hat{k}$	different notations for unit vectors
$\hat{x}$	$\hat{y}$	$\hat{z}$	The "hat" indicates that
$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$	the vector has length 1

A point in space can alternatively be indicated by

$$\bar{r} = r_1 \hat{e}_1 + r_2 \hat{e}_2 + r_3 \hat{e}_3 = \sum_{i=1}^{3} r_i \hat{e}_i = r_i \hat{e}_i$$

(Einstein's notation)

Operations involving vectors (prerequisite)

Sum of vectors

$$\bar{r} = r_i \hat{e}_i \qquad \bar{s} = s_i \hat{e}_i$$

$$\bar{r} + \bar{s} = (r_i + s_i)\hat{e}_i = (r_1 + s_1)\hat{e}_1 + (r_2 + s_2)\hat{e}_2 + (r_3 + s_3)\hat{e}_3$$

Product of a vector times a constant

$$c\,\bar{r} = c\,r_i \hat{e}_i = c\,r_1 \hat{e} + c\,r_2 \hat{e}_2 + c\,r_3 \hat{e}_3$$

Scalar product

$$\bar{r} \cdot \bar{s} = r\,s\cos\vartheta$$

(ϑ: angle between vectors)

$$= r_i s_i = r_1 s_1 + r_2 s_2 + r_3 s_3$$

Vector product

$$\bar{r} \times \bar{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_1 & r_2 & r_3 \\ s_1 & s_2 & s_3 \end{vmatrix}$$

$$= \hat{i}(r_2 s_3 - r_3 s_2) - \hat{j}(r_1 s_3 - r_3 s_1) + \hat{k}(r_1 s_2 - r_2 s_1)$$

Time

In classical mechanics, time is a parameter that has the same value for all observers that hold synchronized clocks, no matter what is their location, velocity and acceleration. This might seem a trivial statement but it is not, time is NOT the same for all observers in relativity.

Reference Frames

In all problems in classical mechanics one needs to choose a reference frame, i.e. an origin for the three cartesian axes and their directions.
Of particular importance are inertial frames of reference, that are frames in which Newton's laws of motion as we learned them in the introductory physics courses are valid.

Once an inertial frame has been identified, any other frame moving at constant velocity with respect to the original frame is also an inertial frame: Newton's laws of motion are valid also in all of these frames, without any change in form.

However a frame that is accelerating or rotating with respect to an inertial frame of reference is NOT an inertial frame: If one wants to use this new frame to describe physics one needs to use Newton's laws of motion which are different with respect to the ones that one employs in inertial frames of reference.

Mass

The mass of an object is a quantitative measure of the resistance that the object offers to being accelerated. Massive objects are not easily accelerated, lighter objects are more easily accelerated.

We know from elementary physics that the weight of an object is proportional to its mass. Therefore a way of comparing masses is to measure their weight (gravitational pull acting on them) when there are placed at the same point in space.

The unit employed to measure mass in the international system is the kilogram.
The definition of the kilogram changed in May 2019, it is now linked to one of the fundamental constants of nature (Plank's constant) rather than to the mass of the standard kilogram, chunk of platinum-iridium alloy stored at the Bureau of Weights and Measures near Paris.

Force

A force is a quantitative measure of a push or a pull received by a physical object. It is a vector, and therefore it is characterized by a magnitude and a direction. The magnitude of the force is measured in the international system in Newtons.

$$N \equiv kg\,\frac{m}{s^2}$$