

Find the stationary path 2

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Problem 6.12 in Taylor

Find the path $y(x)$ that makes the integral below stationary

$$\int_{x_1}^{x_2} \underbrace{x \sqrt{1 - (y')^2}}_{f(y', x)} dx$$

$$\frac{\partial f}{\partial y} = 0 \quad \frac{d}{dx} \frac{\partial f}{\partial y'} = \frac{d}{dx} \left(- \frac{x y'}{\sqrt{1 - (y')^2}} \right) = 0$$

$$- \frac{x y'}{\sqrt{1 - (y')^2}} = c \quad \rightarrow \quad c^2 = (x^2 + c^2)(y')^2$$

$$y' = \frac{c}{\sqrt{x^2 + c^2}} \quad y = \int \frac{c dx}{\sqrt{x^2 + c^2}} + d$$

$$y = \int \frac{dx}{\sqrt{1 + \frac{x^2}{c^2}}} + d$$

$$y = c \int \frac{du}{\sqrt{1 + u^2}} + d = c \operatorname{arcsinh} \left(\frac{x}{c} \right) + d$$